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ON THE ASYMPTOTIC BEHAVIOR OF THE SOLUTIONS OF THE SCHRÖDINCER EQUATION $(-\Delta+Q(y)-k^2)V=F$

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1. Introduction

Let us consider the Schrödinger operator

$$(1.1)\qquad S=-\Delta+Q(y)\qquad (y\in \boldsymbol{R}^N)$$

in $\boldsymbol{R}^N$. The purpose of this work is to show an asymptotic formula for the solution V of the equation $(S-k^2)V=F$ under the assumption that $Q(y)$ is a long-range potential, i.e., $Q(y)=O(|y|^{-\varepsilon})$ $(\varepsilon>0)$ as $|y|\to\infty$. Here $k\in\boldsymbol{R}-\{0\}$ and $F(y)$ is a given function on $\boldsymbol{R}^N$, and the solution V satisfies the "radiation condition"

$$(1.2)\qquad \frac{\partial V}{\partial |y|}-ikV(y)\to 0\qquad (|y|\to\infty).$$

The exact definition of the radiation condition will be given below (Definition 2.1).

Our method has its origin in the works of W. Jäger ([4]~[7]). He considered the differential operator with operator-valued coefficients

$$(1.3)\qquad L=-\frac{d^2}{dr^2}+B(r)+C(r)\qquad r\in I=(0,\infty),$$

where for each $r\in I$ $B(r)$ is a non-negative definite, self-adjoint operator in a Hilbert space X and $C(r)$ is a symmetric operator in X. L acts on X-valued functions on I. In the above papers Jäger, among others, has established the limiting absorption principle for L and an asymptotic formula for the solution v of the equation $(L-k^2)v=f$, which were used to develop an eigenfunction expansion theory associated with L. These results can be applied to the Schrödinger operator as follows: Let $X=L_2(S^{N-1})$, S^{N-1} denoting the $(N-1)$-sphere, and let $L_2(I, X)$ be the Hilbert space of all X-valued functions $F(r)$ on I such that $|f(r)|_X$ is square integralbe on I, where $|\ |_X$ means the norm of X. Then the multiplication operator U of the form