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ON APPROXIMATE SUFFICIENCY

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H. Kudō defined the notion of approximate sufficiency in his paper ([4], [6]) and proved some interesting results. In this paper we obtain some characterizations for it.

1. Notations and definitions

Let $(X, \mathcal{A})$ be a sample space consisting of a set X and a σ-algebra $\mathcal{A}$ of subsets of X. The reader should understand by the word "σ-algebra" and "algebra" a sub-σ-algebra and subalgebra of $\mathcal{A}$, respectively. Given a σ-algebra $\mathcal{B}$ and a finite measure λ on $\mathcal{A}$, $E_\lambda(f\mid\mathcal{B})$ denotes the conditional expectation of a λ-integrable function f over X given $\mathcal{B}$ with respect to λ: *i.e.*, $E_\lambda(f\mid\mathcal{B})$ is a $\mathcal{B}$-measurable function such that $\int_B f d\lambda=\int_B E_\lambda(f\mid\mathcal{B})d\lambda$ for every $B\in\mathcal{B}$. When a probability measure P on $\mathcal{A}$ is absolutely continuous with respect to λ (we write $P\ll\lambda$), $\frac{dP}{d\lambda}$ denotes the Radon-Nikodym derivative. It is clear that $E_\lambda\left(\frac{dP}{d\lambda}\mid\mathcal{B}\right)$ coincides with the Radon-Nikodym derivative $\left.\frac{dP}{d\lambda}\right|_{\mathcal{B}}$ of $P/\mathcal{B}$ with respect to $\lambda/\mathcal{B}$, where $P/\mathcal{B}$ and $\lambda/\mathcal{B}$ are the contractions of P and λ to $\mathcal{B}$ respectively.

For a finite signed measure m, $||m||_{\mathcal{B}}$ denotes the value $\sup_{B\in\mathcal{B}}|m(B)|$. When $m\ll\lambda$ and $m(X)=0$, it is well known that $||m||_{\mathcal{B}}=\frac{1}{2}\int_X\left|\left.\frac{dm}{d\lambda}\right|_{\mathcal{B}}\right|d\lambda$ $\left(=\frac{1}{2}\int_X\left|E_\lambda\left(\frac{dm}{d\lambda}\mid\mathcal{B}\right)\right|d\lambda\right)$. Here and hereafter the integration without any assignment of its domain should be understood as that extended over the whole space X.

Let $\{\mathcal{A}_n\}$ be an increasing sequence of σ-algebras and $\{\mathcal{B}_n\}$ a sequence of σ-algebras satisfying $\mathcal{B}_n\subset\mathcal{A}_n$. According to Kudō ([4], [6]), $\{\mathcal{B}_n\}$ is said to be approximately sufficient for a pair $\{P, Q\}$ of probability measures on $\mathcal{A}$, if for each n there is a pair of probability measures $\{P_n, Q_n\}$ on $\mathcal{A}_n$ such that