Ogawa, S. and Murazawa, T.
Osaka J. Math.
13 (1976), 613-630

ON THE EXISTENCE OF A REPRODUCING KERNEL ON HARMONIC SPACES AND ITS PROPERTIES

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(Received September 10, 1975)

Introduction. Let B be a finite plane domain with the smooth boundary and $\Lambda^2(B)$ the class of all solutions φ of the differential equation $\Delta\varphi-p\varphi=0$ such that

$$D[\varphi]=\iint_B\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2+p\varphi^2\right]dxdy<+\infty\,,$$

where $p=p(x, y)$ is a positive analytic function of real variables x and y in B. S. Bergman [6] proved the existence of a function K which has the characteristic reproducing property of a kernel function, with respect to the Dirichlet integral

$$D[\varphi, \psi]=\iint_B\left[\frac{\partial\varphi}{\partial x}\frac{\partial\psi}{\partial x}+\frac{\partial\varphi}{\partial y}\frac{\partial\psi}{\partial y}+p\varphi\psi\right]dxdy\,.$$

From the point of view of the axiomatic harmonic function theory, B is a space with the pre-sheaf: $U\rightarrow\Lambda^2(U)$, where U is any open subset of B.

The aim of this paper is to show that there exists a reproducing kernel of a space formed by harmonic functions on harmonic spaces in the sense of H. Bauer, to study some properties of the kernel function and to obtain the Cauchy-type representation of harmonic functions by an integral kernel obtained from the reproducing kernel. The results are immediately applicable to the classical harmonic functions on R^n and the family of all solutions of the heat equation on R^{n+1}, and moreover to that of all solutions of more general differential equations on Riemannian manifolds which satisfies Bauer's axioms.

In the paragraph 1, we construct a Hilbert space $R^2(U)$, formed by harmonic functions, with a certain scalar product, and in the paragraph 2, by applying the existence theorem of a kernel function, we discuss that there exists a reproducing kernel of $R^2(U)$. In the paragraph 3, we show the monotonicity of the kernel function with respect to the domain of its definition on harmonic spaces, which is an imprtant property of a class of kernel functions. In the last paragraph, using an integral kernel obtained by the reproducing kernel we study an integral representation of harmonic functions in Cauchy-type.