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ON FINITE HOMOGENEOUS SYMMETRIC SETS

Dedicated to Professor Mutsuo Takahashi on his 60th bitrhday

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1. Introduction

A symmetric set is a set A on which a binary operation $a\circ b$ is defined satisfying the following three axioms:

(1.1) $a\circ a = a$,

(1.2) $(x\circ a)\circ a = x$,

(1.3) $x\circ(a\circ b) = ((x\circ b)\circ a)\circ b$.

The mapping $S_a: A\rightarrow A$ defined by $xS_a = x\circ a$ is a permutation on A by (1.2), and it is called the symmetry around a. Corresponding to the axioms above we have the following:

(1.1′) $aS_a = a$,

(1.2′) $S_a^2 = I$,

(1.3′) $S_{a\circ b} = S_{aS_b} = S_b^{-1}S_aS_b$.

We denote by $G(A)$ the permutation group on A generated by $S_A = \{S_a \mid a\in A\}$. Since $T^{-1}S_aT = S_{aT}$ for $a\in A$ and $T\in G(A)$ by (1.3′), S_A is a set of involutions in $G(A)$ which is $G(A)$-invariant. The subgroup of $G(A)$ generated by $\{S_aS_b \mid a, b\in A\}$ is called the *group of displacements* and is denoted by $H(A)$. The set S_A is a symmetric set with binary operation $S_a\circ S_b = S_b^{-1}S_aS_b$. The mapping $a\mapsto S_a$ of A onto S_A is a homomorphism, and if it is an isomorphism, *i.e.* if $a\neq b$ implies $S_a\neq S_b$ then A is called *effective*. If A is effective then the center $Z(G(A))$ of $G(A)$ is trivial.

REMARK. In [4] and [5] the group of displacements is denoted by $G(A)$.

Now suppose that G is a group and A is a subset of G satisfying the following:

(1.4) A is a set of involutions in G which is G-invariant,

(1.5) G is generated by A.