Yamada, S.
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PIVOTAL MEASURES IN THE CASE OF WEAK DOMINATION

SAKUTARŌ YAMADA

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Weakly dominated statistical structures were introduced by T.S. Pitcher ([6]) and studied by various authors. Notably, D. Mussmann ([5]) proved a generalization of Neyman factorization theorem for sufficient σ-fields.

In this paper we give a construction of the "pivotal measures" based on whose existence his proof is developed. Mussmann's method does not provide for an explicit form of the measure but only an existence-proof of it, as is discussed in detail in the beginning of Section 3. As a result of our method, not only the whole process of arriving at the factorization theorem has been greatly simplified, but simple proofs of some additional results are furnished by making use of this concrete definition of pivotal measures. Moreover, we give a characterization of pivotal measures.

1. Definitions and notations

Let $(\mathcal{X}, \mathcal{A}, \mu)$ be a measure space consisting of a set $\mathcal{X}$, a σ-algebra $\mathcal{A}$ of subsets of $\mathcal{X}$ and a measure μ on $(\mathcal{X}, \mathcal{A})$. We define that $\mathcal{A}_e(\mu)\equiv\{E\in\mathcal{A} \mid \mu(E)<\infty\}$, $\mathcal{A}_{e\sigma}(\mu)\equiv\{A\in\mathcal{A} \mid A=\bigcup_{n=1}^{\infty}E_n,\ E_n\in\mathcal{A}_e(\mu)$ *for all* $n\}$ and $\mathcal{A}_l(\mu)\equiv\{A\subset\mathcal{X} \mid A\cap E\in\mathcal{A}$ *for all* $E\in\mathcal{A}_e(\mu)\}$, μ is said to concentrate on a set T in $\mathcal{A}$ if $\mu(A)=\mu(A\cap T)$ for all A in $\mathcal{A}$.

A family of $\mathcal{A}$-measurable real functions $\{g_E \mid E\in\mathcal{A}_e(\mu)\}$ is called a ***μ-cross section*** ([8]), if each $g_E(x)=0$ outside of E and $I_{E_1\cap E_2}g_{E_1}=I_{E_1\cap E_2}g_{E_2}[\mu]$ for all E_1 and E_2 in $\mathcal{A}_e(\mu)$. Here I_E is the indicator function of E.

μ is called a *localizable* measure if for any family $\mathcal{F}\subset\mathcal{A}_e(\mu)$ there exists an essential supremum of $\mathcal{F}$ in $\mathcal{A}$ with respect to μ, written ess-sup $\mathcal{F}(\mu)$, which is a set in $\mathcal{A}$ satisfying the following two conditions:

(1) $\mu(E-\text{ess-sup}\ \mathcal{F}(\mu))=0$ for all E in $\mathcal{F}$,

(2) $\mu(\text{ess-sup}\ \mathcal{F}(\mu)-A)=0$ for any A in $\mathcal{A}$ satisfying $\mu(E-A)=0$ for all E in $\mathcal{F}$.

μ is called *D-localizable* ([2]) if for any μ-cross section $\{g_E \mid E\in\mathcal{A}_e(\mu)\}$