Date, E.
Osaka J. Math.
12 (1975), 777–793

ON SCATTERING THEORY OF A FIRST ORDER ORDINARY DIFFERENTIAL OPERATOR

ETSURO DATE

(Received January 22, 1975)

In this paper we study the scattering theory of the operator

$$(0.1)\qquad L_u = iID - i\begin{bmatrix} 0 & u_1 & u_2 \\ u_1^* & 0 & 0 \\ u_2^* & 0 & 0 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$
$$D = d/dx\,, \quad u = {}^t(u_1, u_2)$$

where u_1 and u_2 are complex-valued integrable functions, and * denotes complex-conjugate.

The operator L_u has been introduced by Manakov [4] in order to solve the system of non-linear Schrödinger equations

$$(0.2)\qquad iu_t + 2^{-1}u_{xx} + ||u||^2 u = 0\,, \quad ||u||^2 = |u_1|^2 + |u_2|^2$$

in terms of the scattering data of (0.1).

In [4], Manakov has described the scattering theory of (0.1) formally. The scattering theory of (0.1) resembles in many respects to that of the operator

$$(0.3)\qquad L_u = i\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} D - i\begin{bmatrix} 0 & u \\ u^* & 0 \end{bmatrix}$$

which has been introduced by Zakharov and Shabat [7] in order to solve the scalar non-linear Schrödinger equation. The scattering theory of (0.3) has been treated by Tanaka [6] in detail.

The main differences between (0.1) and (0.3) appear in the structure of the scattering matrix (S-matrix) (§2) and in the part of the inverse problem (§5). The S-matrix is the operator which relates the asymptotic behavior of solutions of eigenvalue problem $L_u f = \zeta f$ as $x \to -\infty$ to the asymptotic behavior as $x \to \infty$. As the suitable base of this eigenspace, we take Jost solutions. In the case of (0.3) the S-matrix is 2×2 type and has the form

$$\begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}$$