Hayakawa, K.
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ON THE LIMIT STATE OF SOLUTIONS OF SOME SEMILINEAR DIFFUSION EQUATIONS

KANTARO HAYAKAWA

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Introduction. This paper is concerned with the behavior of solutions of the following Cauchy problem for the semilinear diffusion equation

$$(1) \qquad \begin{aligned} &\partial_t u = \Delta u + f(u)\,, \quad u = u(t, x)\,, \quad t>0, \quad x \in R^N\,, \\ &u(0, x) = u_0(x)\,, \quad x \in R^N\,, \end{aligned}$$

where ∂_t and Δ denote $\partial/\partial t$ and $\sum_{j=1}^{N} \partial^2/\partial x_j^2$.

The type of phenomena that occur to solutions depends of course on the type of the nonlinear term $f(u)$ in the equation. For the function of type $u^{1+\alpha}$, H. Fujita [1] dealt with the problem of blowing-up of solutions in a finite time (see also H. Fujita [2], the present author [3] and S. Sugitani [4]). On the other hand A.M.Kolmogorov-I.G.Petrovsky-N.S.Piscounov [5] and Y.I. Kaneli [6] investigated the behavior of solution $u(t, x)$ of (1) as $t \to \infty$ in the case when the function $f(u)$ is $u(1-u)$ as the typical instance.

Here we deal with the problem (1) for the function of the type $u^{1+\alpha}(1-u)$ and investigate the limit state of the solution $u(t, x)$ as $t \to \infty$. Results may be roughly spoken as follows. Whether all nontrivial solutions tend to 1 or not depends on the degree α of the increase of f near 0. In the latter case solutions tend to 0 or 1 according to the magnitude of the initial value. This seems parallel to results in [1].

Precisely, our results are the followings. We assume that the function $f(r)$ satisfies next conditions (i), (ii) and (iii).

(i) $f(r)$ is of class C^1 on the closed interval [0, 1].
(ii) $f(r)>0$ on the open interval (0,1) and $f(0)=f(1)=0$.
(iii) There exist positive constants C_0 and α, with which we have $f(r) \geqq C_0 r^{1+\alpha}$ for $0 \leqq r \leqq 1/2$.

Further, in Theorem 2, the assumption (iv) should be added.

(iv) $f(r) \leqq C_1 r^{1+\alpha}$ on [0, 1] for some constant $C_1>0$.

For the initial data $u_0(x)$ we only consider such functions that are compatible to $f(r)$, *i.e.*, $0 \leqq u_0(x) \leqq 1$, and that are continuous only for the sake of simplicity.