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ON t-DESIGNS

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Introduction and preliminaries

An *incidence structure* is a triple $S=(X, \mathcal{A}, \mathcal{I})$ where X and $\mathcal{A}$ are disjoint sets and $\mathcal{I}\subseteq X\times\mathcal{A}$. Elements $x\in X$ are called *points* and elements $A\in\mathcal{A}$ are called *blocks* of S. A point x and a block A are *incident* iff $(x, A)\in\mathcal{I}$. For any block A, (A) will denote the set of points incident with A.

Let v, k, t and λ be integers with $v\geq k\geq t\geq 0$ and $\lambda\geq 1$. An $S_\lambda(t, k, v)$ (*a t-design* on v points with block size k and index λ) is an incidence structure $D=(X, \mathcal{A}, \mathcal{I})$ such that

(i) $|X|=v$,
(ii) $|(A)|=k$ for every $A\in\mathcal{A}$,
(iii) for every t-subset T of X, there are exactly λ blocks $A\in\mathcal{A}$ with $T\subseteq(A)$.

It is well known that every $S_\lambda(t, k, v)$ has exactly $b=\lambda\binom{v}{t}\Big/\binom{k}{t}$ blocks and more generally, for any i-subset I of points $(0\leq i\leq t)$, the number of blocks A of the design with $I\subseteq(A)$ is

$$b_i=\lambda\frac{\binom{v-i}{t-i}}{\binom{k-i}{t-i}},$$

independent of the subset I [2].

Abstract: We present the generalization (conjectured by A. Ja. Petrenjuk) of Fisher's Inequality $b\geq v$ for 2–designs and Petrenjuk's Inequality $b\geq\binom{v}{2}$ for 4–designs. The t-designs satisfying the inequality with equality may be considered as generalizations of the symmetric 2–designs $(b=v)$ and have the property that there are exactly $\frac{1}{2}t$ possible values for the size of the intersection of two distinct blocks, these values being computable from the parameters.

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