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ON THE EXCHANGE PROPERTY IN A DIRECTSUM OF INDECOMPOSABLE MODULES

Dedicated to Professor Kiiti Morita on his 60th birthday

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Throughout R will denote a ring with identity and every R-modules considered in this note are unitary R-modules. Let M be an R-module. If $\mathrm{End}_R(M)$ is a local ring, we call M a *completely indecomposable module*. We take a set of completely indecomposable modules $\{M_\alpha\}_I$ and put $M=\sum_I \oplus M_\alpha$. Then we know several properties of M with respect to this decomposition. For instance, let $M=\sum_J \oplus N_\beta$ be another decomposition and I' a finite subset of I, then $M=\sum_{I'} \oplus M_{\alpha'} \oplus \sum_{J-\varphi(I')} \oplus N_\beta$, where φ: $I'\to J$ is a one-to-one into mapping [1]. H. Kanbara [8] shows that the above fact is true for any subset I' of I if and only if $\{M_\alpha\}_I$ is a locally semi-T-nilpotent (see the definition below).

In this note, we fix a subset I' (not necessarily finite) and give criteria for $\sum_{I'} \oplus M_{\alpha'}$ to satisfy the above property. If $\{M_{\alpha'}\}_{I'}$ is locally semi-T-nilpotent, $\sum_{I'} \oplus M_{\alpha'}$ satisfies it, however the converse is not true [4]. When we fix the subset I', the above property does depend not only on $\sum_{I'} \oplus M_{\alpha'}$ but also on $\sum_{I-I'} \oplus M_{\alpha''}$. On the other hand, the concept of semi-T-nilpotency of $\{M_{\alpha'}\}_{I'}$ does depend only on $\sum_{I'} \oplus M_{\alpha'}$. Hence, we shall define a new concept in this note, namely relative semi-T-nilpotency (see the definition below) and give a relation between relative semi-T-nilpotency and the property above.

In the final saction (Appendix), we shall generalize [6], Lemma 5 as Theorem A.1 by virtue of K. Yamagata's idea [12], [13] and [14] (Lemma A.1). That theorem gives the complete proof of [5], Lemma 2 (Corollary 2) and a generalization of [14], Theorem (Theorem A.2).

1. Definition

Let $\{M_\alpha\}_I$ be a set of completely indecomposable modules. We shall recall definitions of locally semi-T-nilpotency and the induced category $\mathfrak{A}$ from