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NON COMMUTATIVE KRULL RINGS

Dedicated to Professor Kiiti Morita for his 60th birthday

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A commutative integral domain D is said to be a Krull domain provided there is a family $\{V_i\}_{i\in I}$ of discrete rank one valuation overrings of D such that

(1) $D=\cap_{i\in I}V_i$.

(2) each V_i is essential for D.

(3) Given x in D, $x\neq 0$, there is at most a finite number of i in I such that x is a non unit in V_i.

Using noetherian local Asano orders and noetherian simple rings instead of discrete rank one valuation rings we will introduce non commutative Krull rings and generalize some elementary results on commutative Krull domains to the case of non commutative Krull rings.

In §1, we will define non commutative Krull rings and study the relations between a prime Goldie ring R and noetherian local Asano orders containing R. We will introduce, in §2, the concept of divisor classes on bounded Krull rings and show that the divisor class of a non commutative Krull ring becomes an abelian group under some conditions. In §3, we will study orders over a commutative Krull domain $\mathfrak{o}$. Maximal $\mathfrak{o}$-orders are bounded Krull rings. Furthermore we will generalize the approximation theorem for commutative Krull domains to the case of maximal $\mathfrak{o}$-orders (Theorem 3.5). In §4 we will define the w and v-operations on one-sided R-ideals of prime Goldie rings in the same fashion as for commutative domains. We will show that these operations coincide on noetherian bounded Krull rings and maximal $\mathfrak{o}$-orders. Further we will show that every class of right v-ideals of maximal $\mathfrak{o}$-orders contains a right ideal generated by two regular elements. Several examples of non commutative Krull rings will be given in the final section.

Throughout this paper R will denote a prime Goldie ring[1] with identity element which is not artinian, and Q will denote the simple artinian quotient ring of R.

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1) Conditions assumed on rings will always be assumed to hold on both sides; for example, a Goldie ring always means a right and left Goldie ring.