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ON $S\underset{R}{\otimes}S$-MODULE STRUCTURE OF S/R-AZUMAYA ALGEBRAS

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Introduction. Let R be a commutative ring and S a commutative R-algebra. An R-Azumaya algebra A is called an *S/R-Azumaya algebra* if A contains S as a maximal commutative subalgebra and is left S-projective. Kanzaki [10] has determined the structure of S/R-Azumaya algebras by using generalized crossed products when S/R is a separable Galois extension. He then has derived directly the so called seven terms exact sequence due to Chase, Harrison and Rosenberg [4], [5]. And recently Hattori [9] has also derived the seven terms exact sequence by another method. In this paper, we shall generalize the notion of cohomology over Hopf algebras introduced by Sweedler [12] and then investigate $S\underset{R}{\otimes}S$-module structure of S/R-Azumaya algebras when S/R is a Hopf Galois extension.

In §1, we shall define the cohomology of algebras over Hopf algebras. Secondly, in §2 we shall give a criterion for S/R-Azumaya algebras to be $S\underset{R}{\otimes}S$-projective. And we shall characterize smash product algebras in §3. Finally we shall give a criterion for $S\underset{R}{\otimes}S$-projective modules to be S/R-Azumaya algebras. In appendix, we shall give a direct proof of the exactness of the following seven terms sequence for an H-Hopf Galois extension S/R;

$$0 \to H^1(H, S/R, U) \to Pic(R) \to H^0(H, S/R, Pic) \to H^2(H, S/R, U) \to$$
$$Br(S/R) \to H^1(H, S/R, Pic) \to H^3(H, S/R, U)$$

where $Br(S/R)$ denotes the Brauer group of R-Azumaya algebras split by S, U denotes the units functor and Pic denotes the Picard group functor.

Throughout, R is a fixed commutative ring with 1. Algebras mean R-algebras, each $\otimes$, Hom, etc. is taken over R unless otherwise stated. Repeated tensor products of an algebra T are denoted by exponents, $T^q = T\otimes\cdots\otimes T$ with q-factors (T^0 means R).

0. Preliminaries

We shall quote for the sake of convenience some definitions, notations and