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# ON THE NUMBER OF THE LATTICE POINTS IN THE AREA $0<x<n,\ 0<y\leqslant ax^k/n$.

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## 1. Introduction

Let $S_a^{(k)}(n)$ be the number of the lattice points in the area $0<x<n$, $0<y\leqslant ax^k/n$, where $k$ and $n$ are positive integers and $a$ is a positive integer which is prime to $n$. Then we have

$$S_a^{(k)}(n)=\sum_{x=1}^{n-1}[ax^k/n]\,,$$

where [ ] denotes the Gauss symbol. Let

$$ax^k/n=[ax^k/n]+\overline{\{ax^k/n\}}\,,$$

where $\overline{\{ax^k/n\}}$ denotes the fractional part of $ax^k/n$. Then we have

$$\sum_{x=1}^{n-1}ax^k/n=S_a^{(k)}(n)+\sum_{x=1}^{n-1}\overline{\{ax^k/n\}}$$

or

$$S_a^{(k)}(n)=\sum_{x=1}^{n-1}ax^k/n-\sum_{x=1}^{n-1}\overline{\{ax^k/n\}}\,.$$

We put

$$S_a^{(k)}(n)=\sum_{x=1}^{n-1}ax^k/n-\frac{n-1}{2}+c_a^{(k)}(n)\,,$$

$$c_a^{(k)}(n)=\frac{n-1}{2}-\sum_{x=1}^{n-1}\overline{\{ax^k/n\}}\,.$$

If we suppose that $S_a^{(k)}(n)$ behaves approximately as $\sum_{x=1}^{n-1}ax^k/n-\frac{n-1}{2}$ then $c_a^{(k)}(n)$ can be regarded as error term. T. Honda has conjectured the followings.

**Conjecture 1.** *For a fixed k and any positive real number $\varepsilon$ we have*

$$c_a^{(k)}(n)=O(n^{((k-1)/k)+\varepsilon})\,,$$

*for $a=1$.*