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ON THE RADICALS OF Γ-RINGS

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1. Introduction

N. Nobusawa [1] introduced the notion of a Γ-ring, more general than a ring, and proved analogues of the Wedderburn-Artin theorems for simple Γ-rings and for semi-simple Γ-rings; Barnes [2] obtained analogues of the classical Noether-Lasker theorems concerning primary representations of ideals for Γ-rings; Luh [3, 4] gave a generalization of the Jacobson structure theorem for primitive Γ-rings having minimum one-sided ideals, and obtained several other structure theorems for simple Γ-rings; Coppage-Luh [5] introduced the notions of Jacobson radical, Levitzki nil radical, nil radical and strongly nilpotent radical for Γ-rings and Barnes' [2] prime radical was studied further. Also, inclusion relations for these radicals were obtained, and it was shown that the radicals all coincide in the case of a Γ-ring which satisfies the descending chain condition on one-sided ideals.

In this paper the notions of semi-prime ideals are extended to Γ-rings, and it is shown that all of the following conditions are equivalent: (1) Q is a semi-prime ideal. (2) Q^c is an n-system. (3) The Γ-residue class ring M/Q contains no non-zero strongly nilpotent ideals. (4) The prime radical $P(Q)$ of the ideal Q coincides with Q. Also, the following characterization of $P(M)$ is obtained. $P(M)$ is a semi-prime ideal which is contained in every semi-prime ideal in M. Let R be the right operator ring of a Γ-ring M. For $P\subseteq R$ and for $Q\subseteq M$ we define $P^*=\{x\in M: [\Gamma, x]\subseteq P\}$ and $Q^{*\prime}=\{\sum_i [\alpha_i, x_i]\in R: M(\sum_i [\alpha_i, x_i])\subseteq Q\}$. In [5] the following theorem was proved. If $P(M)$ is the prime radical of the right operator ring R of the Γ-ring M, then $P(M)=P(R)^*$.

We show the following result dual to the above theorem, $P(R)=P(M)^{*\prime}$. As a result, it is obtained that $P(M)^{*\prime *}=P(M)$ and $P(R)^{**\prime}=P(R)$. The similar properties hold for the Levitzki nil radical and Jacobson radical. Also, some radical properties are cosidered.

2. Preliminaries

Let M and Γ be additive abelian groups. If for all $a,b,c\in M$, and $\alpha,\beta\in\Gamma$,