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# A NOTE ON A FIXED-POINT-FREE AUTOMORPHISM AND A NORMAL P-COMPLEMENT

HIROSI NAGAO AND KAZUMASA NOMURA

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## 1. Introduction

Let $A$ be a group of automorphisms of a group $G$, and denote by $C_G(A)$ the subgroup of $G$ consisting of all the elements fixed by $A$. If $C_G(A)=1$ then $A$ is said to be fixed-point-free. The purpose of this note is to prove the following two theorems.

The first theorem is an extension of a result of F. Gross ([2], Theorem 3.5).

**Theorem 1.** *Let $A$ be a group of automorphisms of a finite group $G$ and $p$ a prime divisor of $|G|$. Suppose that either $A$ is cyclic and fixed-point-free or $(|A|, |G|)=1$ and $C_G(A)$ is a $p$-group. If a Sylow $p$-subgroup $P$ of $G$ is of the form*

$$P = P_1 \times P_2 \times \cdots \times P_l$$

*where $P_i$ is a direct product of $m_i$ cyclic subgroups of order $p^{n_i}$ with $n_1 < n_2 < \cdots < n_l$ and if each $m_i$ is less than any prime divisor of $|A|$, then $G$ has a normal $p$-complement.*

If an abelian $p$-group $P$ is of the form as in the theorem above, we denote $\sum_{i=1}^{l} m_i$ by $m(P)$, and $\max_{1 \leq i \leq l} m_i$ by $\tilde{m}(P)$.

For a $p$-group $P$, $ZJ(P)$ denotes the center of the Thompson subgroup of $P$ and we define $(ZJ)^i(P)$ recursively by the rule

$$(ZJ)^0(P) = 1, \quad (ZJ)^1(P) = ZJ(P), \quad \text{and} \quad ZJ(P/(ZJ)^{i-1}(P)) = (ZJ)^i(P)/(ZJ)^{i-1}(P).$$

In a case of $p$ odd Theorem 1 can be extended as follows.

**Theorem 2.** *Let $G$ be a finite group, $p$ an odd prime divisor of $|G|$ and $P$ a Sylow $p$-subgroup of $G$. Suppose that $G$ has a group $A$ of automorphisms satisfying the same assumption as in Theorem 1. If each $\tilde{m}((ZJ)^i(P)/(ZJ)^{i-1}(P))$ is less than any prime divisor of $|A|$, then $G$ has a normal $p$-complement.*