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K-GROUPS OF SYMMETRIC SPACES I

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Introduction

In this paper we consider the unitary K-groups of compact homogeneous spaces of Lie groups and in particular, we lay emphasis on the compact symmetric spaces.

For a compact connected Lie group G with $\pi_1(G)$ torsion-free as a symmetric space it is known that the K-group $K^*(G)$ of G is an exterior algebra on the elements of $K^{-1}(G)$ induced by the basic representations of G [2], [3], [7]. Making use of this result Hodgkin constructed the Künneth formula spectral sequence in equivariant K-theory [8], [9]. This spectral sequence is our main tool in the present study. Besides we find some examples of the K-groups of symmetric spaces in [5].

The main theorem of this paper is the following

Theorem A. *Let G be a compact connected simply-connected Lie group together with the involutive automorphism σ and K the subgroup of G consisting of fixed points of σ. When we write M for the homogeneous space G/K, we have*

(i) *There are elements $\rho_1, \cdots, \rho_l$ of $R(G)$ such that*

$$\sigma^*(\rho_k) = \rho_k \ (r+1 \leqq k \leqq l) \qquad \textit{for} \quad \textit{some } r \textit{ and}$$
$$R(G) = Z[\rho_1, \cdots, \rho_r, \sigma^*(\rho_1), \cdots, \sigma^*(\rho_r), \rho_{r+1}, \cdots, \rho_l] .$$

(ii) *The natural homomorphism $\alpha\colon Z \underset{R(G)}{\otimes} R(K) \to K^0(M)$ becomes a monomorphism (Section 1) and if we identify an element of $Z \underset{R(G)}{\otimes} R(K)$ with its image by α, then we can write*

$$K^*(M) = \Lambda(\beta(\rho_1 - \sigma^*(\rho_1)), \cdots, \beta(\rho_r - \sigma^*(\rho_r))) \otimes (Z \underset{R(G)}{\otimes} R(K)) .$$

where $\beta(\rho_k - \sigma^*(\rho_k))$ is the element of $K^{-1}(M)$ induced by the representations ρ_k and $\sigma^*(\rho_k)$ in (i) *for $k=1, \cdots, r$ (Section 1).*

(iii) *$K^*(M)$ is torsion-free.*

The arrangement of this paper is as follows.