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ON THE LOOP-ORDER OF A FIBRE SPACE

Dedicated to Professor Ryoji Shizuma on his 60-th birthday

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Introduction

Let ΩX denote the space of loops on a based topological space X. M. Sugawara [8] called the order of the identity class $1_{\Omega X}$ of ΩX in the group $[\Omega X, \Omega X]$ the loop-order of X, denoted by $l(X)$, and proved ([8], Theorem 3) that, for a Hurewicz fibration $F\rightarrow E\rightarrow B$, $l(E)$ is a divisor of the multiple $l(B)\cdot l(F)$.

The aim in this note is to determine, using a technique of Larmore and Thomas [2], the loop-order of a total space obtained as a 2-stage Postnikov tower and to discuss that of a space obtained as a 3-stage Postnikov tower.

In this note, let p denote a fixed prime. Let $\mathcal{A}(p)$ denote the mod p Steenrod algebra, and let ε: $\mathcal{A}(p)\rightarrow\mathcal{A}(p)$ denote the Kristensen map of degree -1, which is a derivation and is given by

$$\varepsilon(Sq^{n}) = Sq^{n-1} \ (n\geqq 1) \qquad \text{if } p=2,$$
$$\varepsilon(\Delta)=1, \quad \varepsilon(P^{k})=0 \ (k\geqq 0) \qquad \text{if } p>2,$$

(cf. [2], Proposition 3.5; [5]). We shall write $\varepsilon(\alpha)=\tilde{\alpha}$.

Also denote by $K_{n}=K(Z_{p}, n)$ the Eilenberg-MacLane complex of type (Z_{p}, n). Let E_{1} and E_{2} be principal fibre spaces with classifying classes

$$\{\theta_{1}, \theta_{2}, \cdots, \theta_{m}\}: K_{n}\rightarrow \mathop{\times}_{j=1}^{m} K_{n+r_{j}}, \qquad 0<r_{1}\leqq r_{2}\leqq\cdots\leqq r_{m}\leqq n-3$$

and

$$\sum_{i=1}^{k}\pi_{i}^{*}\gamma_{i}: \mathop{\times}_{i=1}^{k} K_{n+s_{i}}\rightarrow K_{n+r}, \qquad s_{1}=0\leqq s_{2}\leqq\cdots\leqq s_{k}<r\leqq n-3$$

respectively, where θ_{j} and γ_{i} are cohomology operations of degree r_{j} and $r-s_{i}$, regarded as elements of $\mathcal{A}(p)$, and π_{i}: $\mathop{\times}_{t=1}^{k} K_{n+s_{t}}\rightarrow K_{n+s_{i}}$ is the projection on the i-th factor. We then obtain

Theorem A. *$l(E_{1})=p^{2}$ if, and only if, there exists j, $1\leqq j\leqq m$, such that $\tilde{\theta}_{j}$ does not belong to the left $\mathcal{A}(p)$-module, $\sum_{t=1}^{j-1}\mathcal{A}(p)\theta_{t}$, of $\mathcal{A}(p)$ generated by*