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A NOTE ON THE RELATION OF Z_2-GRADED COMPLEX COBORDISM TO COMPLEX K-THEORY

ZEN-ICHI YOSIMURA

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Let $MU^*(\ )$ and $K^*(\ )$ denote the Z_2-graded complex cobordism theory and the complex K-theory respectively. The Thom homomorphism μ_{c*}: $\pi_0(MU)\to\pi_0(K)$ on coefficient groups is identified (up to sign) with the classical Todd genus Td: $\Lambda\to Z$. We denote by I the ideal of Λ to be the kernel of Td: $\Lambda\to Z$. Wolff [7] proved that the decreasing filtration $\{I^q MU^*(\ )\}$ of $MU^*(\ )$ consists of cohomology theories defined on the category of based finite CW-complexes, and the associated quotients $I^q MU^*(\ )/I^{q+1}MU^*(\ )$ are determined by the complex K-theories $KG_q^*(\ )$ with coefficients $G_q=I^q/I^{q+1}$.

The purpose of this note is to extend the Wolff's result to the category of based CW-complexes. Let F_qMU be the CW-spectrum associated with the cohomology theory $I^qMU^*(\ )$, i.e., $\{Y, F_qMU\}^*\cong I^qMU^*(Y)$ for any based finite CW-complex (or finite CW-spectrum). We show that $\{F_qMU^*(\ )\}$ is a decreasing filtration of $MU^*(\ )$ consisting of Λ-modules so that the associated quotients are equal to $KG_q^*(\ )$, and in addition that $F_{q+1}MU^*(\ )$ is a direct summand of $F_qMU^*(\ )$.

Moreover we give a tower

$$MU\to\cdots\to Q_qMU\to Q_{q-1}MU\to\cdots\to Q_0MU=K$$

of MU-module spectra such that $KG_q\to Q_qMU\to Q_{q-1}MU$ is a cofiber sequence of MU-module spectra, which factorizes the Thom map μ_c: $MU\to K$.

Baas [3] constructed a tower of CW-spectra

$$MU\to\cdots\to MU\langle n\rangle\to MU\langle n-1\rangle\to\cdots\to MU\langle 0\rangle=H$$

factorizing the Thom map μ: $MU\to H$. In appendix we show that the tower is of MU-module spectra and the sequence $\Sigma^{2n}MU\langle n\rangle\xrightarrow{m_{x_n}}MU\langle n\rangle\to MU\langle n-1\rangle$ is a cofiber sequence where m_{x_n} is the multiplication by x_n a ring generator of Λ with degree $2n$.

1. Decreasing filtration of $MU_*(\ )$

1.1. A pair (E,ρ) is called a *Z_2-graded CW-spectrum* if E is a CW-spectrum