Kawauchi, A,
Osaka J. Math.
12 (1975), 565–581

THREE DIMENSIONAL HOMOLOGY HANDLES AND CIRCLES

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(Received December 10, 1974)

This paper will extend the known propertes of the Alexander polynomials of classical knot complements to the properties of the Alexander polynomials of arbitrary (possibly non-orientable) compact 3-manifolds with infinite cyclic first homology groups. In particular, the Alexander polynomial will always have a reciprocal property. The existence of the corresponding manifolds and the other related results will be shown.

1. Statement of results

Throughout this paper, spaces will be considered in the PL category.

DEFINITION 1.1. A compact 3-manifold M is called a *homology orientable handle* if M has the homology of an orientable handle: $H_*(M; Z) \approx H_*(S^1 \times S^2; Z)$. Likewise, M is a *homology non-orientable handle* if $H_*(M; Z) \approx H_*(S^1 \times_\tau S^2; Z)$, a *homology orientable circle* if $H_*(M; Z) \approx H_*(S^1; Z)$ and $\partial M = S^1 \times S^1$, and a *homology non-orientable circle* if $H_*(M; Z) \approx H_*(S^1; Z)$ and $\partial M = S^1 \times_\tau S^1$.

It is easily seen that if M is a homology orientable (or non-orientable, respectively) handle or circle then M is orientable (or non-orientable, respectively) as a manifold. [Note that, in case $\partial M \neq \phi$, $H_3(M, \partial M; Z) \approx H_2(\partial M; Z)$.]

By $\mathcal{C}(S^1 \times S^2)$, $\mathcal{C}(S^1 \times_\tau S^2)$, $\mathcal{C}(S^1 \times B^2)$ and $\mathcal{C}(S^1 \times_\tau B^2)$, we denote the class of homology orientable handles, the class of homology non-orientable handles, the class of homology orientable circles and the class of homology non-orientable circles, respectively.

The following Theorem 1.2 implies that a compact connected 3-manifold M with $H_1(M; Z) = Z$ belongs to one of the four classes $\mathcal{C}(S^1 \times S^2)$, $\mathcal{C}(S^1 \times_\tau S^2)$, $\mathcal{C}(S^1 \times B^2)$ and $\mathcal{C}(S^1 \times_\tau B^2)$ if ∂M contains no 2-spheres.

Theorem 1.2. *Let M be a compact connected 3-manifold with $H_1(M; Z) = Z$. If $\partial M = \phi$, then $H_*(M; Z)$ is isomorphic to either $H_*(S^1 \times S^2; Z)$ or $H_*(S^1 \times_\tau S^2; Z)$. If $\partial M \neq \phi$, then under the assumption that ∂M contains no 2-spheres, $H_*(M; Z) \approx H_*(S^1; Z)$ and ∂M is homeomorphic to either $S^1 \times S^1$ or $S^1 \times_\tau S^1$.*

If ∂M contains a 2-sphere, then we will attach a 3-cell to eliminate it. This