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UNIPOTENT ELEMENTS AND CHARACTERS OF FINITE CHEVALLEY GROUPS

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Introduction

Let © be a connected semisimple linear algebraic group defined over an algebraically closed field K of characteristic $p>0$, and σ a surjective endomorphism of © such that the group $\mathfrak{G}_\sigma$ of elements fixed by σ is finite. The finite groups $\mathfrak{G}_\sigma$ obtained in this manner can be classified as follows (Steinberg [20]): If © is simple, $\mathfrak{G}_\sigma$ is either the group of rational points of a F-form of © for an appropriate finite field F or one of the groups defined by M. Suzuki and R. Ree. If © is not simple, $\mathfrak{G}_\sigma$ is essentially a direct product of the groups mentioned above.

In this paper, a finite group G is called a finite Chevalley group[1] if it can be realized as $\mathfrak{G}_\sigma$ for some © and σ. Let (G, B, N, S) be a Tits system (or BN-pair) associated to a finite Chevalley group G. We denote by W its Weyl group. Let G^1 be the set of unipotent elements (or p-elements) of G and U the p-Sylow subgroup of G contained in B. The main purpose of this paper is to establish the following two results:

(I) *Let w be an arbitrary element of W, and w_S the element of W of maximal length. Then the number of unipotent elements contained in the double coset BwB is* $|BwB\cap w_S U w_S^{-1}|\,|U|$, *which can be written explicitly as a polynomial in* $q_s=|BsB/B|\,(s\in S)$[2]. (As a corollary, we obtain $|G^1|=|U|$, a result of Steinberg [20].)

(II) *Assume that the characteristic p is good* (see Definition 6.2) *for ©. Let g be an element of $G=\mathfrak{G}_\sigma$, and C a regular unipotent conjugacy class of G. Then the number $|Bg\cap C|$ depends neither on g nor C.*

As far as the author knows, these results are new even for $G=SL_n(F)$ with F a finite field. In this case an arbitrary prime is good and a unipotent element

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1) This definition is slightly different from the one given, for example, in [19]. But such difference is not essential for our purpose.

2) For a finite set A, $|A|$ denotes the number of its elements.