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ON TIGHT 4-DESIGNS

To the memory to Otto Grün*

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1. Introduction

Let v, k and λ be positive integers with $v>k$. Let X be a v-set and *Jl* a family of k-subsets of X. *(X, Jl)* is called a 4-(v, k, λ) design (or simply a 4-design) if for each 4-subset T of X there exist precisely λ elements of *Jl* containing T. By a theorem of Fisher-Petrenjuk [2] the number of elements in $\mathcal{A}$ is not less than $\frac{1}{2}v(v-1)$. If it is equal to $\frac{1}{2}v(v-1)$, *(X, Jl)* is called tight.

If $v\geqq 6$ and if *Jl* is the family of all $(v-2)$-subsets of X, *(X, Jĭ)* is a tight 4-design. Such tight 4-designs are called trivial.

Let *(X, Jl)* be a 4-design. If $v-k\geqq 4$ and if $\mathcal{A}^c$ is the family of $(v-k)$-subsets of X each of which is a complement of an element of $\mathcal{A}$ in X, *(X, Jlc)* is a 4-design. *(X, Jl)* and *(X, Jlc)* are called complementary with each other. Furthermore if *(X, Jl)* is tight, *(X, Jlc)* is also tight.

There exist only two known non-trivial tight 4-designs *(X, Jl)* they are a 4-(27, 7, 1) design and a 4-(23, 16, 52) design. They are complementary with each other. We call these designs Witt tight designs, because they are found by Witt [5], [6].

Now the purpose of this paper is to prove the following theorem.

Theorem. *Let (X, Jl) be a non-trivial tight* 4-(v, s, λ) *design. Then (X, Jl) is a Witt tight design.*

Our proof relies on the following theorem of Wilson and Ray-Chaudhuri [4]: Let *(X,Jl)* be a tight 4-(v, k, λ) design. Then a non-negative integer μ is called an intersection number of $(X,\mathcal{A})$, if there exist two distinct elements A and B of *Jl* such that $|A\cap B|=\mu$. There exist precisely two intersection numbers, say, μ_1 and μ_2, where $\mu_2>\mu_1$ μ_1 and μ_2 are the roots of the polynomial

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