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A CHARACTERIZATION OF THE TRIANGULAR MATRIX RINGS OVER QF RINGS

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In [4], Harada proved that a ring R is a right QF-3 and semi-primary hereditary ring if and only if R is a direct sum of rings whose basic rings are the rings of triangular matrices over division rings. We consider an analogous result to the above one for a right QF-3 semi-primary ring with some injective properties. By Zaks [8], the ring R of triangular matrices of degree $n\geqq 2$ over a QF ring has the injective dimension one both as right and left R-modules, and moreover it is easy to see that the ring R is a QF-3 ring whose maximal right quotient ring is a QF ring. It is our purpose to show that for a basic indecomposable semi-primary ring, the converse is also true.

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Throughout this paper we shall assume that every ring R has an identity element 1, and every R-module is unitary. The notations M_R and ${}_RM$ are used to underline the fact that M is a right or a left R-module, respectively. For a ring R, a right (resp. left) R-module M is called a *minimal faithful module* if M is a faithful R-module and every faithful right (resp. left) R-module contains an isomorphic image of M as a direct summand. A ring R is called a *right (resp. left) OF-3 ring* if R has a minimal faithful right (resp. left) module, and R is called a *QF-3 ring* if R is both a right and left QF-3 ring.

For a semi-primary ring R, the following conditions are equivalent, (see Jans [5]).

(1) R is a right QF-3 ring.

(2) R has a faithful projective injective right ideal.

Let S be a ring which contains a ring R as a subring. Then S is called a *right (resp. left) quotient ring* of R if S is a rational extension of R as a right (resp. left) R-module. By R' we denote the maximal right quotient ring of R. If R is a QF-3 ring, then the maximal right quotient ring of R coincides with the maximal left quotient ring of R, (see Tachikawa [7]).

Lemma 1. *Let S be a QF ring. Let R be a right QF-3 ring such that R*