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GALOIS THEORY AND IDEALS IN COMMUTATIVE RINGS*

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Introduction. Let A be a commutative ring with 1. For any ideal i of A, let $r(i)$ denote the radical of i, which is the set of all elements of A some power of which lies in i or, equivalently, the intersection of all prime ideals of A which contain i. If p is a prime ideal of A and X is a unital A-module, let X_p denote the module of fractions of X with respect to the complement of p in A. If q is a primary ideal of A; then $p=r(q)$ is a prime ideal of A and the ideal-length of q, denoted by $\lambda(q)$, is the length of a composition series for the A-module $A_p/q\,A_p$.

In the sequel, let B be a given commutative ring with 1, let G be a given finite group of automorphisms of B, and let A be the subring of G-invariant elements of B. For any prime ideal p in A, G is represented as a group of automorphisms of B_p by the formula $\sigma\left(\frac{b}{s}\right)=\frac{\sigma(b)}{s}$ for $\sigma\in$ G, $b\in B$, and $s\in A-p$ and A_p is the subring of G-invariant elements of B_p by [4, Chap. V, §1, Prop.

Abstract. Let B be a commutative ring with 1, Let G be a finite group of automorphisms of B, and let A be the subring of G-invariant elements of B. For an A-subalgebra A' of B, which has the property that A'_p is a separable A_p-algebra for every prime ideal p in A, the following assertions are proved. If q is a primary ideal of A; then $q\,A'$ has a unique irredundant representation as a finite intersection of primary ideals of A', each primary component of $q\,A'$ lies over q and has ideal-length equal to the ideal-length of q, and the associated prime ideals of $q\,A'$ are the prime ideals of A' which lie over the radical of q. If, in addition, A' is G-stable, then the contraction map is an isomorphism of the lattice of G-stable ideals of A' onto the lattice of ideals of A. Also it is demonstrated that there exists a maximal A-subalgebra A' of B which has the property that A'_p is a separable A_p-algebra for every prime ideal p in A, and this subalgebra is unique and G-stable.

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