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EXAMPLES OF FOLIATIONS WITH NON TRIVIAL EXOTIC CHARACTERISTIC CLASSES

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Introduction

An example of foliation of codimension one with non trivial Godbillon-Vey invariant ([3]) was constructed by R. Roussarie (see Bott [1]). Generalizing the Godbillon-Vey invariant, R. Bott [1] has defined exotic characteristic classes for foliations. In this paper, we shall construct examples of foliations with non trivial exotic characteristic classes.

Roussarie's example was constructed on a compact quotient space of $SL(2;\boldsymbol{R})$ by a discrete subgroup. This example may be regarded as an Anosov foliation arising from the geodesic flow on the unit tangent sphere bundle of a surface with constant negative curvature. This suggests us to consider such a foliation on the unit tangent sphere bundle of a closed $(q+1)$-manifold $(q\geqq 1)$ with constant negative curvature. In fact, our example is constructed as follows. Let G denote the identity component of the Lie group

$$O(q+1,\ 1) = \{X\in GL(q+2;\boldsymbol{R});\ {}^tXBX=B\}\ ,$$

where

$$B=\begin{pmatrix} I_{q+1} & 0 \\ 0 & -1 \end{pmatrix}.$$

Consider a compact subgroup

$$H=\left\{\begin{pmatrix} X & 0 \\ 0 & I_2 \end{pmatrix};\ X\in SO(q)\right\}$$

of G, and a closed subgroup K consisting of $X=(x_{ij})\in G$ such that

$$\det\begin{pmatrix} x_{q+1\,q+1} & x_{q+1\,q+2} \\ x_{q+2\,q+1} & x_{q+2\,q+2} \end{pmatrix} = 1$$

and

$$x_{i\,q+1}+x_{i\,q+2}=0\ (i=1,\cdots,q)\,.$$