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ON FIXED POINT FREE INVOLUTIONS OF T^3

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1. Introduction. In 1962, Y. Tao [4] proved the following theorem in this *Osaka Mathematical Journal*:

Theorem. *If h is a fixed point free involution of $S^1\times S^2$, and if M is the 3-manifoldobtained by identifyingx and hx in S^1 X S^2, then M is either homeomorphic to* (1) $S^1\times S^2$, *or* (2) *the 3-dimensional Klein Bottle, or* (3) $S^1\times P^2$, *or* (4) $P^3\# P^3$.

In order to prove the theorem, Tao used a result of Livesay [2] and simple cut and paste techniques. The question naturally arises as to whether or not Tao's method can be applied to classify the orbit spaces of fixed point free involutions on any manifold of the form $S^1\times F$, where F is a compact surface. We answer this question affirmatively in the case when F is the 2-dimensional torus T^2. In particular, we shall show that if h is a fixed point free involution on the 3-dimensional torus $T^3=S^1\times T^2$, then pasting the points equivalent under h, we must obtain either T^3, or $S^1 X K^2$, or K^3, or the torus bundle over S^1 obtained from [0,1] X T^2 by identifying the boundaries with a homeomorphism h of period two such that $h(m)=m^{-1}$ and $h(l)=ml$, where *(m, /)* is a meridian-longitude system for T^2.

2. **Preliminaries.** The interior of a topological mainfold M will be denoted by *int M* and the boundary by ∂M. The n-dimensional sphere, torus and Klein bottle will be denoted by S^n, T^n, and K^n, respectively.

Since we may assume [3] that T^3 has a fixed triangulation and that h acts piecewise linearly on this triangulation, the objects in this paper (maps, neighborhoods, simple closed curves, etc.) should always be considered from the polyhedral point of view.

We shall think of T^3, K^3 and $S^1 X K^2$ as obtained from [0,1] X T^2 by identifying 0 X T^2 with 1 X T^2. Thus, if (m, l) denotes a meridian-longitude pair for T^2 and $m_i=i\times m$, $l_i=i\times l (i=0,1)$, then identifying $0\times T^2$ with 1 x T^2 so that m_0, l_0 gets glued onto m_1, l_1, respectively, results in a manifold homeomorphic to T^3. For K^3 we must identify m_0, l_0 with m_1^{-1} and l_1^{-1}, respectively, and for S^1 X K^2, m_0, l_0 identifies with m_1^{-1} and l_1, respectively.