Landweber, P.S.
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$BP_*(BP)$ AND TYPICAL FORMAL GROUPS

PETER S. LANDWEBER[1]

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1. Introduction. D. Quillen showed in [6] that the formal group law of complex cobordism is a universal formal group, hence for a commutative ring R there is a natural bijection between ring homomorphisms $MU_*\rightarrow R$ and formal groups over R, where MU_* is the coefficient ring of complex cobordism. Similarly, S. Araki [4] has shown that for a fixed prime p, the formal group law of Brown-Peterson cohomology is universal for typical group laws over commutative $Z_{(p)}$-algebras. Thus if R is a commutative $Z_{(p)}$-algebra, there is a natural bijection between ring homomorphisms $BP_*\rightarrow R$ and typical formal groups over R, where BP_* is the coefficient ring of Brown-Peterson cohomology.

In this note we shall show that $BP_*(BP)$ represents isomorphisms between typical formal groups over $Z_{(p)}$-algebras. This places $BP_*(BP)$ in a purely algebraic setting, as was done for $MU_*(MU)$ in the Appendix to [5]. We show how the structure maps for $BP_*(BP)$ arise in this context, and use our point of view to derive the formulas of J.F. Adams [2, Theorem 16.1] for these structure maps.

All this works as well for $MU_*(MU)$, by omitting mention of *typical* formal groups; this gives a description of $MU_*(MU)$ which is somewhat different from the one given in [5]. In the BP-case it is essential to use coordinates for curves over a typical formal group μ which depend on μ. But in the MU- case, it is optional whether one uses "moving coordinates" (as we do here) or "absolute coordinates" as in [5].

The ideas in this note grew out of musings over D. Ravenel's paper [7] on multiplicative operations in $BP^*(BP)$.

2. **Recollections** (Araki [3, §1] and [4]) For the most part we follow Araki's notation. All rings and algebras are to be commutative. By an *isomor-*

Abstract. It is shown that $BP_*(BP)$ represents the functor which assigns to a commutative $Z_{(p)}$- algebra R the set of isomorphisms between typical formal groups over R. The structure maps of the Hopf algebra $BP_*(BP)$ all arise naturally from this point of view, and one can easily derive the formulas of Adams [2, Theorem 16.1] for them.

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