MULTIPLICATIVE OPERATIONS IN BP COHOMOLOGY

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Introduction. In the present work we study multiplicative operations in BP cohomology. In §1 we show that all multiplicative operations in $BP^*$ are automorphisms (Theorem 1.3). Thus they from the group $\text{Aut}(BP)$. In §2 we define Adams operations in $BP^*$ by the formal group $\mu_{BP}$ of BP cohomology and study the basic properties of them. These operations are primarily defined for units in $\mathbb{Z}_p$ and then extended to $p$-adic units. Thereby we discuss $BP^*$ by extending the ground ring $\mathbb{Z}_p$ to the ring of $p$-adic integers $\mathbb{Z}_p$. To achieve this extension simply by tensoring with $\mathbb{Z}_p$ we restrict our cohomologies to the category of finite CW-complexes. Correspondingly we consider all multiplicative operations in $BP^*(\mathbb{Z}_p)\otimes\mathbb{Z}_p$ whenever it becomes necessary to do so. Adams operations could be defined also for non-units, but we are not interested in such a case in this paper. In §3 we prove that the center of $\text{Aut}(BP)$ consists of all Adams operations (Theorem 3.1).

We regard the lecture note [2] as our basic reference and use the results contained therein rather freely.

1. Multiplicative operations in $BP^*$.

Let $BP^*$ denote the Brown-Peterson cohomology for a specified prime $p$. By a multiplicative operation in $BP^*$ we understand a stable, linear and degree-preserving cohomology operation

\[\Theta_a: BP^*(\ ) \to BP^*(\ )\]

which is multiplicative and $\Theta_a(1) = 1$. The set of all multiplicative operations in $BP^*$ forms a semi-group by composition, which will be denoted by $\text{Mult}(BP)$.

With respect to the standard complex orientation of $BP^*$ [1], [2], [7], we denote by $e_{BP}(L)$ the Euler class of a complex line bundle $L$ and by $\mu_{BP}$ the associated formal group. Let $\Theta_a \in \text{Mult}(BP)$. Putting

\[\Theta_a(e_{BP}(L)) = \sum_{i \in \mathbb{Z}} \theta_i(e_{BP}(L))^i\]

for an arbitrary line bundle $L$, by naturality we obtain a well-determined power