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MULTIPLICATIVE OPERATIONS IN *BP* COHOMOLOGY

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Introduction. In the present work we study multiplicative operations in *BP* cohomology. In § 1 we show that all multiplicative operations in BP^* are automorphisms (Theorem 1.3). Thus they from the group Aut *(BP)*. In §2 we define Adams operations in BP^* by the formal group μ_{BP} of *BP* cohomology and study the basic proprties of them. These oprations are primarily defined for units in $Z_{(p)}$ and then extended to p-adic units. Thereby we discuss BP^* by extending the ground ring $Z_{(p)}$ to the ring of$p$-adic integers Z_p. To achieve this extension simply by tensoring with Zp we restrict our cohomologies to the category of finite CW-complexes. Correspondingly we consider all multiplicative operations in $BP^*(\ \ )\otimes Z_p$ whenever it becomes necessary to do so. Adams operations could be defined also for non-units, but we are not interested in such a case in this paper. In §3 we prove that the center of Aut *(BP)* consists of all Adams operations (Theorem 3.1).

We regard the lecture note [2] as our basic reference and use the results contained there rather freely.

1. Multiplicative operations in BP^*.

Let BP^* denote the Brown-Peterson cohomology for a specified prime p. By a *multiplicative* operation in BP^* we understand a stable, linear and degree-preserving cohomology operation

$$\Theta_a: BP^*(\ \ ) \rightarrow BP^*(\ \ ) \tag{1.1}$$

which is multiplicative and $\Theta_a(1)=1$. The set of all multiplicative operations in BP^* forms a semi-group by composition, which will be denoted by Mult (*BP*).

With respect to the standard complex orientation of BP^* [1], [2], [7], we denote by $e^{BP}(L)$ the Euler class of a complex line bundle L and by μ_{BP} the associated formal group. Let $\Theta_a\in$Mult *(BP)*. Putting

$$\Theta_a(e^{BP}(L)) = \sum_{i\geq 0} \theta_i(e^{BP}(L))^i$$

for an arbitrary line bundle L, by naturality we obtain a well-determined power