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THE HEAT EQUATION ON COMPACT LIE GROUP

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Introduction

McKean and Singer [9] posed the problem of the existence of an analogue of the Poisson's summation formula for manifolds other than flat tori. Y. Colin de Verdiere [3] gave an answer to it in the case of a 2-dimensional compact Riemannian manifold with negative sectional curvature.

The purpose of this paper is to determine the Minakshisundaram's expansion (Theorem 3) related to the heat equation and to give an analogue of the Poisson's summation formula of the fundamental solution of this equation on a simply-connected compact semi-simple Lie group (Theorem 2).

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1. Preliminaries

Let (M, g) be an n-dimensional compact connected orientable Riemannian manifold with the fundamental tensor g, and Δ be the Laplace-Beltrami operator acting on differentiable functions of M. Consider the *fundamental solution* $Z(t, m, m')$ of the heat equation

$$\Delta_x u(t, x) = \frac{\partial}{\partial t} u(t, x).$$

Then it satisfies the following facts:

(i) $0 < Z(t, m, m') \in C^\infty((0, \infty) \times M \times M)$

(ii) $\dfrac{\partial Z}{\partial t} = \Delta_m Z = \Delta_{m'} Z$

(iii) $\lim_{t \to 0+} \int_M Z(t, m, m') f(m') dv_{m'} = f(m), \qquad m \in M$

for every continuous function f on M where $dv_{m'}$ is the volume element of (M,g). And also define the trace $Z(t)$ of $Z(t, m, m')$ as

$$Z(t) = \int_M Z(t, m, m) dv_m. \tag{1.1}$$