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A NOTE ON A RESULT OF KAMAE*

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In [3] Kamae proved that for each constant c, there exists a finite string y such that

$$K(y)-K(y\,|\,x)>c$$

for all but finitely many finite strings x, where $K(\ )$ and $K(\ \backslash\ \cdot)$ are the unconditional and conditional minimal-program complexity measures respectively of Kolmogorov [4]. By considering infinite sequences we are able to obtain a slightly stronger statement of this result.

Let X^∞ denote the set of all infinite binary strings. For $x\in X^\infty$ let x^n denote the initial segment of x of length n, i.e., $x^n=x(1)\cdots x(n)$. To simplify matters we will associate with each finite binary string y the integer n whose binary representation is $1y$. By this means we will consider complexity expressions of the form $K(x^n)$, $K(x^n\,|\,m)$ and $K(m)$ for $x\in X^\infty$ and integers n and m. Then $K(x^n\,|\,n)$ is Kolmogorov's restricted conditional complexity. By a recursively enumerable sequence we mean the characteristic sequence of a recursively enumerable set. By "$\overset{\infty}{\exists} n$" and "$\overset{\infty}{\forall} n$" we mean respectively "There exist infinitely many integers n" and "For all but finitely many integers n."

Theorem. *There exists a recursively enumerable sequence x, such that*

$$\forall c\overset{\infty}{\exists} n\overset{\infty}{\forall} m.\quad K(x^n\,|\,n)-K(x^n\,|\,m)>c\,.$$

Proof. We need the following two lemmas.

Lemma. *For every recursively enumerable sequence x,*

$$\exists c_1\forall n\overset{\infty}{\forall} m.\quad K(x^n\,|\,m)\leq K(n)+c_1\,.$$

Proof. Let h be a total recursive function which enumerates the 1's of x, i.e., $x(i)=1\Leftrightarrow\exists j.\,h(j)=i$. Define the total recursive function $f(n,m)$ as follows:

Step 1: Enumerate via h the first m 1's of the sequence x, i.e. compute $h(1),\cdots,h(m)$.

Step 2: Output the following finite string y of length n— $y(i)=1\Leftrightarrow i$ appears on

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