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ON THE ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF CERTAIN NON-AUTONOMOUS DIFFERENTIAL EQUATIONS

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1. Introduction

In this paper conditions are obtained under which all solutions of certain real non-autonomous nonlinear differential equations tend to zero as $t\to\infty$.

Theorem 1 is concerned with the system of differential equations;

(1.1) $\quad \& = A(t)x+f(t,x)$

where x, f are n-dimensional vectors, $A(t)$ is a bounded continuously differentiable $n\times n$ matrix for $t\geqq 0$, and $f(t, x)$ is continuous in (t, x) for $t\geqq 0$, $||x||<\infty$, here $||\ ||$ denotes the Euclidean norm.

Theorem 2 is concerned with the differential equation of the third order;

(1.2) $\quad \dddot{x}+a(t)f(x,\dot{x},\ddot{x})\ddot{x}+b(t)g(x,\dot{x})+c(t)h(x)=p(t, x, \dot{x}, \ddot{x})$

where $a(t)$, $b(t)$, $c(t)$ are positive continuously differentiable and /, g, h, p are continuous real-valued functions depending only on the arguments shown, and the dots indicate the differentiation with respect to t.

The asymptotic property of solutions of third order differential equations has received a considerable amount of attention during the past two decades, particularly when (1.2) is autonomous. Many of these results are summarized in [11].

A few authors have studied non-autonomous third order differential equations. K. E. Swick [13] considered the following equations

(1.3) $\quad \dddot{x}+p(t)\ddot{x}+q(t)g(\dot{x})+r(t)h(x)= 0$,

(1.4) $\quad \dddot{x}+f(x, \dot{x}, t)\ddot{x}+q(t)g(\dot{x})+r(t)h(x)= 0$,

with the assumption that $q(t)$, $r(t)$ are positive, bounded and monotone decreasing.

In [6], the author studied the asymptotic behavior of the solutions of the equation