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ON THE HITTING PROPERTIES OF A CLASS OF ONE-DIMENSIONAL MARKOV PROCESSES

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1. Introduction. Let $X=(X_t, P_x, x\in R^1)$ be a one-dimensional standard Markov process with generator A

$$Au(x) = a(x)u'(x)+\sigma(x)^2u''(x)/2 + \int_{-\infty}^{\infty}\left\{u(x+y)-u(x)-\frac{y}{1+y^2}u'(x)\right\}n(x, dy)\,. \tag{1.1}$$

In this article, we shall discuss how the sample paths of X approach a single point. Let σ_0 be the first hitting time of the sample path to the origin: $\sigma_0=\inf\{t>0; X_t=0\}$. Set $\Omega_1=\{\omega; \sigma_0(\omega)<+\infty\}$. Define Ω_1^+, Ω_1^- and $\Omega_1^\pm$ by

$$\Omega_1^+ = \{\omega\in\Omega_1;\ \exists\varepsilon>0,\ \forall t\in[\sigma_0(\omega)-\varepsilon,\sigma_0(\omega)),\ X_t(\omega)>0\}\ ,$$
$$\Omega_1^- = \{\omega\in\Omega_1;\ \exists\varepsilon>0,\ \forall t\in[\sigma_0(\omega)-\varepsilon,\sigma_0(\omega)),\ X_t(\omega)<0\}\ ,$$

and

$$\Omega_1^\pm = \{\omega\in\Omega_1;\ \exists t_n\uparrow\sigma_0(\omega)\quad s.t.\ X_{t_{2n-1}}(\omega)<0<X_{t_{2n}}(\omega)\}\ .$$

Our present problem is to decide whether $P_x(\Omega_1^+/\Omega_1)$, $P_x(\Omega_1^-/\Omega_1)$ and $P_x(\Omega_1^\pm/\Omega_1)$ are positive or not. When X is spatially homogeneous, the problem was treated by T.Takada [15] in case $\sigma\neq 0$ and by N.Ikeda-S.Watanabe [3] in case $\sigma=0$ who also applied their results to the study of two-dimensional diffusion processes. Their method is based on the estimate of the singularity of the Green function on the diagonal set. Recently P.W.Millar [10] solved a similar problem independently. Let T_x be the first exit time of the sample path of a spatially homogeneous process from $(-\infty, x]$ $(x>0)$. Millar gave, in terms of the exponent, a necessary and sufficient condition that $P_0(X_{T_x}=x)>0$.

Here we shall consider the class of spatially inhomogeneous Markov processes determined by A under certain regularity conditions on a, σ and Lévy measure $n(x, dy)$. We shall give some sufficient conditions that $P_x(\Omega_1^+/\Omega_1)=1$, $P_x(\Omega_1^-/\Omega_1)=1$ and $P_x(\Omega_1^\pm/\Omega_1)=1$. Our method consists in estimating the singularity of the Green function as in [3]. More precisely, under the regularity conditions that will be given in §2, put