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CONTINUOUS MAPS OF MANIFOLDS WITH INVOLUTION III

MINORU NAKAOKA

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Introduction

Let N and M be m-dimensional closed manifolds on each of which an involution T is given, and let $f:N\to M$ be a continuous map. In the preceding paper [7], on the assumption that the involutions T of M and N are both free the author introduced a mod 2 integer $\hat{\chi}(f)$ called the equivariant Lefschetz number of f, and proved that if $\hat{\chi}(f)\not\equiv 0$ then f has an equivariant point. In this paper the result will be generalized to the case when the involution T of N is not necessarily free.

The former result was proved through the use of the equivariant point index $\hat{I}(f)$, which is constructed from the class $\Delta_\infty\in H^m(S^\infty\underset{T}{\times}M^2)$ requiring that the involution T of N is free (see [7]). Taking in place of $S^\infty\underset{T}{\times}M^2$ the pair of the symmetric product of M and its diagonal, we define a new equivariant point index $\hat{I}(f)$ provided that the involution of M is free and the involution of N is non-trivial. The new result will be proved by making use of the new index.

Recently, to show that certain homotopy classes in closed manifolds cannot be realized by embedded sphere, R. Fenn [5] has proved a theorem of the Borsuk-Ulam type. In this paper, $\hat{I}(f)$ will be also used to generalize the Fenn theorem.

Throughout this paper, the homology and cohomology with coefficients in $\boldsymbol{Z}_2$ are to be understood.

1. Preliminaries

Let N be a compact polyhedron with PL(=piecewise linear) involution T. We denote by F the fixed point set of T, and by N_T the quotient of N with respect to T. Let $\pi\colon N\to N_T$ be the projection, and put $F_T=\pi(F)$. The following facts are well known (see [1], [2], [6], [9], [10]).

There are the transfer homomorphism

$$\phi^*\colon H^q(N)\to H^q(N_T,\ F_T)$$

and the Smith homomorphism