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ON THE BP_*-HOPF INVARIANT

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In this paper we will consider the BP_*-Hopf invariant, $\pi_*(S^0)\to \mathrm{Ext}^{1,*}_{BP_*(BP)}(BP_*, BP_*)$, i.e. the Hopf invariant defined by making use of the homology theory of the Brown-Peterson spectrum BP. The BP_*-Hopf invariant is essentially "the functional coaction character". Similarly we will define the BP_*-e invariant ("the functional Chern-Dold character") and show that the BP_*-Hopf invariant coincides with the BP_*-e invariant by the BP-analogue of Buhstaber-Panov's theorem ([6], [7]). As applications we give a proof of the non-existence of elements of Hopf invariant 1, and detect α-series.

We will use freely notations of Adams [2], [3], [4]. For example, S, H, HZ_p and $HZ_{(p)}$ denote the sphere spectrum, the Eilenberg-MacLane spectrum, Z_p coefficient Eilenberg-MacLane spectrum and $Z_{(p)}$ coeffitient Eilenberg-MacLane spectrum respectively, where $Z_{(p)}$ is the ring of integers localized at the fixed prime p.

We list some well known facts:

$$\pi_*(BP)=BP_*(S^0)=BP_*=Z_{(p)}[v_1, v_2, \cdots], \quad \deg v_k=|v_k|=2(p^k-1).$$
$$H_*(BP)=HZ_{(p)*}(BP)=Z_{(p)}[n_1, n_2, \cdots], \quad \deg n_k=|n_k|=2(p^k-1).$$

The Hurewicz map

$$h^H=(i^H\wedge 1_{BP})_*: \pi_*(BP)\to H_*(BP)$$

is decided by the formula [5]

$$h^H(v_k)=pn_k-\sum_{0<s<k} h^H(v_{k-s})^{p^s}n_s.$$
$$BP_*(BP)=BP_*[t_1, t_2, \cdots], \quad \deg t_k=|t_k|=2(p^k-1).$$

The Thom map $BP\overset{\mu}{\to} HZ$ induces

$$BP_*(BP)\overset{\mu}{\to} HZ_{(p)*}(BP)=H_*(BP), \quad \mu(t_k)=n_k; \quad \mu(v_k\cdot 1)=0$$

$(k>0)$ and ([10])