Takeuchi, Y.
Osaka J. Math.
12 (1975), 23-31

ON GALOIS OBJECTS WHICH ARE STRONGLY RADICIAL OVER ITS BASIC RING

Dedicated to Professor M. Takahashi on his 60 th birthday

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(Received March 7, 1974)

In [3], Chase and Sweedler introduced a notion of Galois object and extended, in this case, the fundamental theorem of Galois theory for fields. Furthermore, they showed that it contains, as a special case, the fundamental theorem of Galois theory on separable algebras developed by Chase, Harrison and Rosenberg. However they mentioned in [3] that they had, in general, no good characterization of the subalgebras which arised in the Galois correspondence. A purpose of this paper is to show what those subalgebras are in the case of strongly radicial extensions. On the other hand, it is well-known that for a finite purely inseparable extension K of a field k, there exists a chain of subfields of K: $K=K_0\supset K_1\supset K_2\supset\cdots\supset K_r=k$ such that K_i is of exponent one over K_{i+1} for $i=0, 1, 2, \cdots, r-1$. We shall study this analogy in the case of Galois objects over a field which are strongly radicial over their basic field.

Let H be a finite cocommutative split[1] Hopf algebra over a commutative ring A and C a Galois H^*[2]-object over A which is strongly radicial over A.

In our first section, we shall study a coalgebra structure of H. Moreover we shall show that there exists a bijection between the set of admissible[3] Hopf subalgebras of H and the set of distinguished[4] intermediate rings between A and C.

In our second section, we shall exhibit an existence of a sequence of subrings of C

$$C=C_0\supset C_1\supset\cdots\supset C_i\supset\cdots\supset C_n=A$$

satisfying the followings for each $i=0, 1, \cdots, n-1$:

(1) $C\# K_i\cong Hom_{C_i}(C, C)$ via a canonical map where K_i is a Hopf subalgebra of H.

(2) $d(C_i)\subseteq C_i$ for $d\in H$.

(3) $C_i[\mathcal{D}er_1(C_i/C_{i+1})]=Hom_{C_{i+1}}(C_i, C_i)$ for $i=0, 1, \cdots, n-1$.

1) For the definition, see §1.
2) H^* denotes a dual Hopf algebra of H.
3) For the definition, see [3, Def. 7.1].
4) For the definition, see a following part of the proof of Proposition 3 below.