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ON MULTIPLY TRANSITIVE GROUPS XII

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1. Introduction

The known 4-fold transitive groups are the symmetric groups S_n $(n\geq 4)$, the alternating groups A_n $(n\geq 6)$ and Mathieu groups M_n $(n=11, 12, 23, 24)$. The main purpose of this paper is to characterize these known 4-fold transitive groups. The result is as follows.

Theorem. *Let G be a 4-fold transitive group on $\Omega=\{1, 2, \cdots, n\}$. Assume that*

(*) *t is the maximal number of fixed points of involutions of G.*

Furthermore assume that G contains a 2-subgroup Q which satisfies the following conditions:

(1) *$|I(Q)|=t$ and Q is a Sylow 2-subgroup of $G_{I(Q)}$,*

(2) *$N(Q)^{I(Q)}=S_t$ or A_t.*

Then G is one of the following groups; S_n $(n\geq 4)$, A_n $(n\geq 6)$ or M_n $(n=11, 12, 23, 24)$.

This theorem is a generalization of theorems of M. Hall ([2], Theorem 5.8.1), H. Nagao [10] and the author [11]: the case $t<4$ has been proved by M. Hall, the case $t=4$ or 5 by H. Nagao and the case $t=6$ or 7 and $N(Q)^{I(Q)}=A_t$ by the author.

The followings are corollaries.

Corollary 1. *Let G be a 4-fold transitive group on $\Omega=\{1, 2, \cdots, n\}$, and P a Sylow 2-subgroup of a stabilizer of four points in G. Assume that n is even and $P\neq 1$.*

(1) *If $I(P)=I(Z(P))$, where $Z(P)$ is the center of P, then G is one of the following groups; S_n $(n\geq 6)$, A_n $(n\geq 8$ and $n\equiv 0$ (mod 4)) or M_{12}.*

(2) *For any point i of $\Omega-I(P)$ if P_i is semiregular $(\neq 1)$ on $\Omega-I(P_i)$ or 1, then G is one of the following groups; S_6, S_8, A_8, A_{10}, M_{12} or M_{24}.*

Corollary 2. *Let G be a 4-fold transitive group on $\Omega=\{1, 2, \cdots, n\}$ and P a Sylow 2-subgroup of a stabilizer of four points in G. If P is a transitive group*