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ON STRONGLY INVARIANT COEFFICIENT RINGS

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Let A and B be rings with an identity. After P. Eakin and W. Heinzer we shall say that A and B are stably equivalent if there is an integer n such that polynomial rings $A[X_1, \cdots, X_n]$ and $B[Y_1, \cdots, Y_n]$ are isomorphic [3]. A number of recent investigations have been published concerning the study of this equivalence. One of the interesting question is the one, called the cancellation problem for rings, which asks when the stably equivalence implies the isomorphism. A ring with this property will be called an invariant ring. This paper contains some contribution to this problem.

In §1 we shall take up the notions of strongly invriant rings which are defined by several authors in their own way and make it clear the relationship among them. In §2 we shall consider the following problem: Let A be a strongly invariant ring. What conditions on A guarantees the invariance of the polynomial ring $A[X]$? Several conditions will be given. In the last section we shall give some examples of strongly invariant rings which have not such a nice property. As a result we can give examples of non invariant rings which are two dimensional affine domains over a field of positive characteristic.

1. Strongly invariant rings

In this paper $A[X_1, \cdots, X_n]$ and $B[Y_1, \cdots, Y_n]$ denote always polynomial rings in n indeterminates $X_1, \cdots, X_n$ and $Y_1, \cdots, Y_n$ over rings A and B, respectively. In this paper all rings are assumed to have an identity.

1.1. DEFINITIONS. (i) A ring A is said CE-strongly invariant provided that an isomorphism of rings $\sigma; A[X_1, \cdots, X_n] \to B[Y_1, \cdots, Y_n]$ yields $B[Y_1, \cdots, Y_n] = B[\sigma(X_1), \cdots, \sigma(X_n)]$ ([2]).
(ii) A ring A is said AHE-strongly invariant provided that an isomorphism of rings $\sigma; A[X_1, \cdots, X_n] \to B[Y_1, \cdots, Y_n]$ yields $\sigma(A) = B$ [1].

Let A be a commutative ring. We denote by $N(A)$ the nil radical of A and let A_{red} be the reduced ring $A/N(A)$. Let $R = A[X_1, \cdots, X_n]$. Then $N(R) = N(A)[X_1, \cdots, X_n]$ and $R_{\text{red}} = A_{\text{red}}[\bar{X}_1, \cdots, \bar{X}_n]$ where $\bar{X}_i \equiv X_i \pmod{N(R)}$. If $\sigma; A[X_1, \cdots, X_n] \to B[Y_1, \cdots, Y_n]$ is an isomorphism, then $\sigma(N(A)[X_1, \cdots, X]) = N(B)[Y_1, \cdots, Y_n]$. Hence we have the induced isomorphism $\bar{\sigma}; A_{\text{red}}[\bar{X}_1, \cdots, \bar{X}_n] \to$