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ON MULTIPLY TRANSITIVE PERMUTATION GROUPS III

EIICHI BANNAI*

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Introduction

The purpose of the present note is to prove the following theorem.

Theorem 1. *Let p be an odd prime $\geqslant 11$. Then there exists no permutation group G on a set $\Omega=\{1, 2, \cdots, n\}$ which satisfies the following two conditions:*
(i) *G is $(p+1)$-ply transitive, and $n\equiv p \pmod{p^2}$, and*
(ii) *the order of $G_{1,2,\cdots,p+1}$, the stabilizer of $p+1$ points of Ω in G, is not divisible by p.*

This Theorem 1 is a kind of (but not full even in the case of $p\geqslant 11$) generalization of the result (Theorem 1) in [1]. In the case of $p\geqslant 11$, the Main Theorem in [2] (i.e., the determination of $2p$-ply transitive permutation groups whose stabilizer of $2p$ points is of order prime to p) is also completed alternatively by combining this Theorem 1 with the result of Miyamoto [6].

The brief outline of the proof of Theorem 1 is as follows. The proof will be done by the way of contradiction. First we will show that the symmetric group S_p is not involved in the group $GL(p-3, p)$, if $p\geqslant 11$ (Theorem A). This is proved by the similar argument as in [1, §1], by exploiting the (ordinary, modular and projective) representation theories of the symmetric groups. Next, we will restrict the structure of the Sylow p subgroup P_0 of $G_{1,2,\cdots,p}$. That is, if $|P_0|>p^{p+1}$ then we have $|Z(P_0)|>p$, and moreover we can lead a contradiction by using the well known theorem of Burnside on fusion of elements in the center of a sylow p subgroup and by using a consequence (Theorem B) of Theorem A. If $|P_0|\leqslant p^{p+1}$, then we can show (also by using Theorem A) that we have only one possibility for the structure of P_0, namely, P_0 is isomorphic to the extraspecial p group of order p^{p+1} and of exponent p. Finally, we exclude this remaining case and complete the proof of Theorem 1. This is done by considering the fusion of p elements in P_0. The proof of this final step was provided by T. Yoshida.

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The present address: The Ohio State University, Columbus, Ohio, U.S.A.