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MODULES OVER DEDEKIND PRIME RINGS IV

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Throughout this paper, R will denote a Dedekind prime ring with the quotient ring Q. Let F be any non-trivial right additive topology. A short exact sequence $0\to L\to M\to N\to 0$ is said to be F^∞-pure if the induced sequence $0\to L_F\to M_F\to N_F\to 0$ is splitting exact, where M_F is the F-torsion submodule of M. A right R-module is said to be F^∞-pure injective if it has the injective property relative to the class of F^∞-pure exact sequences. Similarly we shall define the concept of F^∞-pure projective modules.

We have already determined the structures of F^∞-pure injective and F^∞-pure projective modules, under some conditions for F.

In this paper, we shall show how the results in [4] on these injectivity and projectivity can be carried over the case of modules over any topology, and discuss the relationships between F^∞-pure injective modules and F-injective modules. We shall show, in Theorem 2.2, that there is a duality between all F-reduced, F-torsion-free, F^∞-pure injective modules and all F-torsion, F-injective modules. This is a generalization of a theorem of Harrison [2]. By using the duality we shall give some properties of F-torsion-free and F^∞-pure injective modules.

1. F^∞-pure injective modules

Let R be a Dedekind prime ring with the two-sided quotient ring Q. We denote the (R, R)-bimodule Q/R by K. Let F be a non-trivial (right additive) topology. Then we denote the left additive topology corresponding to F by F_l (cf. [5]). For any module M, we denote the F-torsion submodule of M by M_F. Let $Q_F=\varinjlim I^{-1}(I\in F)$. Then $Q_F=\varinjlim J^{-1}$ $(J\in F_l)$ and $K_F=Q_F/R=K_{F_l}$ (cf. [5]). In this paper, F is a fixed non-trivial topology. Following [7], a module D is *F-injective* if $\mathrm{Ext}(R/I, D)=0$ for every $I\in F$. For any module M, we denote the injective hull of M by $E(M)$ and denote the F-injective hull of it by $E_F(M)$. A module G is *F-cotorsion* if $\mathrm{Ext}(Q_F, G)=0$. Let M be a module. The union of all F_l-divisible submodules of M is itself F_l-divisible and will be denoted by MF^∞: if $MF^\infty=0$, then M will be said to be *F-reduced*. From the exact sequence