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Osaka J. Math.
11 (1974), 503–516

THE COMPLEX BORDISM OF CYCLIC GROUPS

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(Received January 22, 1974)
(Revised June 10, 1974)

Introduction. In their book, *Differentiable Periodic Maps* [2], P.E. Conner and E.E. Floyd initiated the study of cobordism groups of periodic maps and succeeded in determining the additive structure of the cobordism groups of free orientation-preserving $\boldsymbol{Z}_p$-actions on manifolds for odd primes p and of free $\boldsymbol{Z}_p$-actions preserving a stably almost-complex structure for arbitrary primes by calculating $MSO_*(B\boldsymbol{Z}_p)$ and $MU_*(B\boldsymbol{Z}_p)$ respectively. Kamata [5] obtained the same results for $MU_*(B\boldsymbol{Z}_p)$ using slightly different methods. We extend these results to a determination of $MU_*(BG)$ where G is an arbitrary cyclic group. The main result is Proposition 16:

$$MU_{2n+1}(B\boldsymbol{Z}_{p^s}) \cong \sum_{a=1}^{s} \sum_{b=p^{a-1}-1}^{n} \frac{\Gamma_{2(n-b)}(p^a)}{p^{\left[\frac{b-p^{a-1}+1}{p^{a-1}(p-1)}\right]+s-a+1}\Gamma_{2(n-b)}(p^a)}$$

where $\Gamma_*(p^a) \simeq MU_*/\langle CP(p-1)^{p^{a-1}}\rangle$ and the square brackets indicate the greatest integer function. We show this by constructing an explicit set of generators coming from the K-theory of the generalized lens spaces $L^n(p^s)$ and computing the order of the group they generate.

I would like to thank the referee for catching several embarrassing errors and suggesting ways of correcting them.

Results. We will have need of several homology and cohomology theories. Following J.F. Adams, let H be the Eilenberg-MacLane spectrum for the integers, K the BU spectrum, and MU the Thom spectrum for the unitary group. The resulting homology theories are denoted by $H_*(\ )$, $K_*(\ )$, and $MU_*(\ )$, and similarly in the case of cohomology theories. When we have need of unreduced theories, we write X^+ for the disjoint union of X and a basepoint, so that $H_*(X^+)$, for example, is ordinary, unreduced, integral homology. In dealing with K-theory, we will be exclusively concerned with $K^0(X)$ which we agree to write as $K(X)$, remembering that this is the reduced group, i.e., what is usually written

1) This work was partially supported by an NSF Graduate Fellowship.