Ochiai, T.
Osaka J. Math.
11 (1974), 483-501

SOME REMARK ON THE DEFECT RELATION OF HOLOMORPHIC CURVES

TAKUSHIRO OCHIAI[1]

(Received December 24, 1973)

1. Introduction. Since R. Nevanlinna established the value distribution theory for meromorphic functions in 1925 ([7]), many extensive works have been done for its generalization in one way or another. One of the far reaching generalization was given by H. Cartan ([3]), H. and J. Weyl ([9]) and L.V. Ahlfors ([1]), which is well-known as a classical theory on the defect relation of holomorphic curves. When we formulate their theory in the relatively new language of the holomorphic line bundles as was done originally by S.S. Chern, we strongly suspect that still further development should be possible. However no substantial progress has been made yet beyond their result. Therefore the author believes that it is of some use to give certain result in this direction though it is rather direct from the classical theory. Thus the purpose of this paper is to explain it in somewhat self-contained manner.

Let $F: \boldsymbol{C} \rightarrow \boldsymbol{P}^n$ be a holomorphic mapping where $\boldsymbol{C}$ is the complex line and $\boldsymbol{P}^n$ is the n-dimensional complex projective space. We assume F to be *non-degenerate* in the sense that the image $F(\boldsymbol{C})$ does not belong to a hyperplane. Then for each hyperplane Φ we can define a *defect* $\delta_F(\Phi)$, having the properties; 1) $0 \leqq \delta_F(\Phi) \leqq 1$; 2) $\delta_F(\Phi)=1$ if $F(\boldsymbol{C}) \cap \Phi$ is empty. Roughly speaking $\delta_F(\Phi)$ measures how often $F(\boldsymbol{C})$ intersects with Φ. Then for a set of hyperplanes $\Phi_j (1 \leqq j \leqq q)$ in general position, we have

$$\sum_{j=1}^{q} \delta_F(\Phi_j) \leqq n+1 .$$

The above is a very brief outline of the classical theory. Now we remark that the set of all hyperplanes is the complete linear system of divisors given by the hyperplane bundle over $\boldsymbol{P}^n$. Then we are ready to consider the following situation. Let M be a connected compact complex manifold and $\boldsymbol{L}$ be a holomorphic line bundle over M. Let V be a linear subspace of the space $\Gamma(M, \boldsymbol{L})$ of all holomorphic cross-sections of $\boldsymbol{L}$. Each non-zero element ϕ in V defines

1) This work was partially supported by the Sakkokai Foundation, Tokyo, Japan.