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LINEAR $SU(n)$-ACTIONS ON COMPLEX PROJECTIVE SPACES

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0. Introduction

Let U_* be the bordism ring of weakly complex manifolds and let G be a compact Lie group. Denote by $SF(G)$, an ideal in U_* of those bordism classes represented by a weakly complex manifold on which the group G acts smoothly without stationary points and the action preserves a weakly complex structure.

For a compact abelian Lie group G the ideal $SF(G)$ was computed by tom Dieck [8]. Such ideals are similarly defined in the bordism ring Ω_* of oriented manifolds and those were computed for certain abelian groups by Floyd [3] and Stong [7]. But it seems that there is no useful method to compute the ideal $SF(G)$ for a non-abelian Lie group G.

First we give an upper bound and a lower bound of $SF(G)$ for any compact Lie group G. To state our result precisely we introduce some notations as follows. Denote by $I(G)$, a set of positive integers such that $n \in I(G)$ if and only if there is an n-dimensional complex G-vector space without G-invariant one-dimensional subspaces, by $m(G)$ the maximum dimension of proper closed subgroups of G, and put

$$n(G) = \dim G - m(G) .$$

It is known that the bordism ring $U_* = \sum_{k \geqslant 0} U_{2k}$ is generated by a set of bordism classes

$$\{[P_n(\boldsymbol{C})],\ [H_{p,q}(\boldsymbol{C})];\ n \geqslant 0,\ p \geqslant q > 0\}$$

as a ring. Now we define ideals $L(G)$, $M(G)$ in U_* as follows. Let $L(G)$ be an ideal in U_* generated by a set

$$\{[P_n(\boldsymbol{C})],\ [H_{m+n,n}(\boldsymbol{C})];\ n+1 \in I(G),\ m \geqslant 0\}$$

and let

$$M(G) = \sum_{2k \geqslant n(G)} U_{2k} .$$

Then we have following results.