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ORIENTED BORDISM MODULES OF S^1- AND $(Z_2)^k$-ACTIONS

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Introduction

In [2] P.E. Conner and E.E. Floyd demonstrated the effectiveness of bordism methods in the studies of group actions. Afterwards, using the bordism methods, many topologists obtained various results in the area. The central tools in these studies are the bordism modules of group actions.

Let G be a compact Lie group, and $\mathfrak{F}$, $\mathfrak{F}'$ be families of subgroups of G such that $\mathfrak{F}\supset\mathfrak{F}'$. We may define the oriented bordism module $\Omega_*(G; \mathfrak{F}, \mathfrak{F}')$, over the oriented cobordism ring Ω_*, which consists of bordism classes of $(\mathfrak{F}, \mathfrak{F}')$-free oriented G-manifolds. In this paper we are concerned with the module structure of $\Omega_*(G; \mathfrak{F}, \mathfrak{F}')$. If $\mathfrak{F}'$ is empty, then we denote this module by $\Omega_*(G; \mathfrak{F})$. Let $\mathfrak{F}_A$ be the family of all subgroups of G. Then $\Omega_*(G; \mathfrak{F}_A)$ is the bordism module of all closed oriented G-manifolds. Especially we are interested in the module structure of $\Omega_*(G; \mathfrak{F}_A)$.

R.E.Stong [7] has shown that $\Omega_*(G; \mathfrak{F}_A)$ is a free Ω_*-module on even dimensional generators when G is a finite p-primary abelian group for odd prime p. Recently E.R.Wheeler [8] has shown that $\Omega_*(G; \mathfrak{F}_A)\otimes R_2$ is a free $\Omega_*\otimes R_2$-module on even dimensional generators when G is a certain finite cyclic group, where $R_2=Z\left[\frac{1}{2}\right]$.

We study the cases in which G is the circle group S^1 or $(Z_2)^k=Z_2\oplus\cdots\oplus Z_2$ (k times). We obtain that both $\Omega_*(S^1; \mathfrak{F}_A)\otimes R_2$ and $\Omega_*((Z_2)^k; \mathfrak{F}_A)\otimes R_2$ are free $\Omega_*\otimes R_2$-modules on even dimensional generators. In fact we prove such "freeness" theorems for more general families, as stated in Theorem 2–1–1 and Theorem 3–1–1.

Our main tools are the Conner-Floyd exact sequences and the fact that $\Omega_*(G; \mathfrak{F}, \mathfrak{F}')$ can be interpreted as (direct sum of) singular bordism modules of adequate spaces when $\mathfrak{F}-\mathfrak{F}'$ consists of a single element H. When G is S^1 or $(Z_2)^k$, this interpretation involves a difficulty for the sake of non-orientability of normal bundles of H-stationary point sets. We overcome this difficulty by a modification of the methods due to E. Ossa [5; Lemma 1–2–5] [6; Lemma 4], (see Lemmas 2–2–3, 3–2–4).