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ON THE FIRST MAIN THEOREM OF HOLOMORPHIC MAPPINGS FROM C^2 INTO $Q_{n-1}(C)$

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0. Introduction

Let f be a holomorphic mapping of a complex line $\boldsymbol{C}$ into a complex projective space $P_n(\boldsymbol{C})$ and suppose that the image $f(\boldsymbol{C})$ is not contained in any hyperplane of $P_n(\boldsymbol{C})$. Put $V[t]=\{z\in\boldsymbol{C}: \log|z|<t\}$, and for a hyperplane ξ in $P_n(\boldsymbol{C})$ let $n(t, \xi)$ be the number of points in $V[t]\cap f^{-1}(\xi)$. Let Ω be the colsed form of degree 2 associated with the Fubini-Study metric on $P_n(\boldsymbol{C})$ and normalized as $\int_{P_n}\Omega^n=1$. The counting function $N(r, \xi)$ and the order function $T(r)$ being defined by

$$N(r, \xi)=\int_0^r n(t, \xi)dt\,, \tag{0.1}$$

$$T(r)=\int_0^r dt\int_{V[t]}f^*\Omega \tag{0.2}$$

respectively, the following equation is known as the First Main Theorem:

$$N(r, \xi)+(m(r, \xi)-m(0, \xi))=T(r)\,, \tag{0.3}$$

where $m(r, \xi)$ is a non-negative function defined for $r\in\boldsymbol{R}^+$ and hyperplanes ξ in $P_n(\boldsymbol{C})$. The term $(m(r, \xi)-m(0, \xi))$ is called the compensating term. It follows from the equation (0.3) that the image $f(\boldsymbol{C})$ intersects with almost all hyperplanes in $P_n(\boldsymbol{C})$. Furthermore it is known that the number of hyperplanes in general position not intersecting with $f(\boldsymbol{C})$ is at most $n+1$. These results are originally due to Ahlfors, and treated also by H. Wu [6] and S. S. Chern [1] in a modernized form.

Let f be a holomorphic mapping of $\boldsymbol{C}^2$ into a complex quadratic $Q_{n-1}(\boldsymbol{C})$ ($n\geqq 3$) satisfying certain non-degenerate conditions [§2]. We consider $Q_{n-1}(\boldsymbol{C})$ as a fixed hypersurface in $P_n(\boldsymbol{C})$. We consider a special family of $(n-2)$-dimensional projective spaces $P_{n-2}(\boldsymbol{C})$ in $P_n(\boldsymbol{C})$ parametrized by a Grassmann manifold $G(\boldsymbol{R})$ of 2-dimensional linear spaces in $\boldsymbol{R}^{n+1}$ [§1]. This family determines a family of $(n-3)$-dimensional complex quadratic $\xi_\alpha(\alpha\in G(\boldsymbol{R}))$ in $Q_{n-1}(\boldsymbol{C})$, each of whose elements is a Poincaré dual of the form Ω^2 in $Q_{n-1}(\boldsymbol{C})$.