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ON THE EXISTENCE OF CHARACTERS OF DEFECT ZERO

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1. Introduction

Let G be a finite group of order g. Let p be a prime and let $g=p^a g'$ with $(p, g')=1$. An irreducible (complex) character of G is called p-defect zero if its degree is divisible by p^a. The following problem is still open (see Feit [6]).

What are some necessary and sufficient conditions for the existence of characters of p-defect zero?

In [15] we, have tried somewhat ring theoritical approaches to the problem (see also Iizuka and Watanabe [11]). Now, since a character of p-defect zero constitutes a p-block for itself, having the identity group as its defect group, we have the following consequences from the theory of defect groups of blocks. Namely if G posseses a character of p-defect zero, then

1. (Brauer [2]) *G contains an element of p-defect zero, i.e. one which is commutative with no non-trivial p-element of G.*
2. (Brauer [2]) *G contains no non-trivial normal p-subgroup.*
3. (Green [8]) *There exist two Sylow p-subgroups S,T of G such that $S\cap T=\{1\}$.* (*This implies the second assertion above*)

Furthermore the Theorem of Clifford shows that if G possesses a character of p-defect zero, then

4. (Clifford-Schur) *Every proper normal subgroup possesses a character of p-defect zero.*

Of course, the above four conditions are not sufficient in general for the existence of a character of p-defect zero (e.g. $G=A_7$, the alternating group on seven letters, $p=2$ or $p=3$).

However, in [12] Ito showed that if G is solvable and has an element of p-defect zero which is contained in $O_{p'}(G)$, the maximal normal p'-subgroup of G, then G possesses a character of p-defect zero. Also in [13] he showed that under certain circumstances the second condition implies the existence of a