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# ON MULTIPLY TRANSITIVE PERMUTATION GROUPS I

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## Introduction

In [5], M. Hall determined 4-ply transitive permutation groups whose stabilizer of 4 points is of odd order. (See also Nagao [11].) On the other hand, in Bannai [1] and Miyamoto [9], $t$-ply transitive finite permutation groups in which the stabilizer of $t$ points is of order prime to an odd prime $p$ have been determined for $t=p^2+p$ and $3p$ respectively. The purpose of this seies of notes is to strengthen those results. In this first note, we will improve Lemma 2.1 in Miyamoto [9]. Namely, we will prove the following result.

**Theorem 1.** *Let $p$ be an odd prime. Then there exists no permutation group $G$ on a set $\Omega=\{1, 2, \cdots, n\}$ which satisfies the following three conditions:*
(i) *$G$ is $(p+2)$-ply transitive, and $n\equiv 2 \pmod p$,*
(ii) *a Sylow $p$ subgroup $P_0$ of $G_{1,2,\cdots,p+2}$ is semiregular on $\Omega-\{1, 2, \cdots, p+2\}$, and*
(iii) $|P_0|\geqslant p^2$.

**Corollary to Theorem 1.** *Let $p$ be an odd prime. Let $G$ be a $(2p+2)$-ply transitive permutation group on a set $\Omega=\{1, 2, \cdots, n\}$. If the order of $G_{1,2,\cdots,2p+2}$ is not divisible by $p$, then $G$ must be $S_n(2p+2\leqslant n\leqslant 3p+1)$ or $A_n(2p+4\leqslant n\leqslant 3p+1)$.*

This corollary is immediately proved by combining Theorem 1 with a result of Miyamoto [9]. To be more precise, if the order of $G_{1,2,\cdots,p+2}$ is not divisible by $p^2$, then the $2p$-ply transitive group $G_{1,2}$ on $\Omega-\{1, 2\}$ must contain $A^{\Omega-\{1,2\}}$ by the result of Miyamoto [9, §1], and so $G$ must be one of the groups listed in the conclusion of the corollary. If the order of $G_{1,2,\cdots,p+2}$ is divisible by $p^2$, then the $(p+2)$-ply transitive group $G_{1,2\cdots,i}$ on $\Omega-\{1, 2, \cdots, i\}$ (if $n\equiv i+2 \pmod p$ with $0\leqslant i\leqslant p-1$) satisfies the three conditions of Theorem 1, and we have a contraidction.

In our proof of Theorem 1, the following result is very important. This result is a kind of generalization of a result of Jordan [8, Chap. IV], and will be of independent interest.

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