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ON A CHARACTERIZATION OF A CLASS OF THE REGULAR GRAPHS OF DIAMETER 2

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1. Introduction

S.S. Schrikhande [2], [3], W.S. Conner [4] and A.J. Hoffman [5] determined all of the graphs with intersection matrices $B(2t-2, t-2, 2)$ for $t \geqq 2$ and $B(2t-4, t-2, 4)$ for $t \geqq 4$.

The lattice graphs of dimension 2 with intersection matrices $B(2t-2, t-2, 2)$ for $t \geqq 3$ and the triangle graphs with intersection matrices $B(2t-4, t-2, 4)$ for $t \geqq 6$ have the remarkable property that for any three vertices which are not joined to each other, no vertex is joined to all of these three vertices.

The purpose of this paper is to prove the following.

Theorem 1. *Let Γ be the regular graph of diameter 2 satisfying the following conditions:*

1. *For any two vertices which are joined to each other, the number of the vertices joined to them is constant.*
2. *There exist three vertices which are not joined each other.*
3. *For any three vertices which are not joined to each other, no vertex is joined to all of these three vertices.*

Then, Γ is one of the following graphs:

a) *L_2-graphs, that is, the lattice graphs of dimension 2 with intersection matrices $B(2t-2, t-2, 2)$ for $t \geqq 3$.*

b) *T_2-graphs, that is, the triangle graphs with intersection matrices $B(2t-4, t-2, 4)$ for $t \geqq 6$.*

c) *the graph defined by 27 lines on cubic surface with intersection matrix $B(16, 10, 8)$.*

d) *L_2'-graphs, that is, the graphs such that for any two vertices which are not joined to each other, the number of the vertices joined to them is one or two. The number of the vertices of the graph is $5 \cdot h^2$ for $h \geqq 2$, and the graph exists uniquely for any integer $h \geqq 2$.*