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CERTAIN INVARIANT SUBRINGS ARE GORENSTEIN II

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Introduction

Let $R=k[X_1, \cdots, X_n]$ be a polynomial ring over a field k and G be a finite subgroup of $GL(n, k)$ with $(|G|, ch(k))=1$, if $ch(k)\neq 0$. We want to investigate the problem; "When is the invariant subring R^G Gorenstein?" The main result of this paper is the following theorem.

Theorem 1. *We assume that G contains no pseudo-reflections. Then R^G is Gorenstein if and only if $G\subset SL(n, k)$.*

Recall that $g\in GL(n, k)$ is a pseudo-reflection if $\text{rank}(g-I)=1$ and g has a finite order (where I denotes the unit matrix). It is known that R^G is again a polynomial ring if and only if G is generated by its pseudo-reflections (cf. [7], Théorème 1). So it would be natural to assume that G contains no pseudo-reflections.

The "if" part was treated in [13]. So, in this paper, we consider the "only if" part. To achieve the proof, we need the theory of the canonical module of a Macaulay ring developed in [2]. As R^G is a Macaulay ring, it has the canonical module K_{R^G}, which is unique up to isomorphisms. R^G is Gorenstein if and only if $K_{R^G}\cong R^G$. We want to construct a canonical module of R^G. In this case, as R^G is normal, a canonical module is isomorphic to a divisorial ideal of R^G. Thus the canonical module K_{R^G} determines a well-defined class $c(K_{R^G})$ of the divisor class group $C(R^G)$ of R^G. R^G is Gorenstein if and only if $c(K_{R^G})=0$. But by the "Galois descente" theory of divisor class groups, $C(R^G)\cong \text{Hom}(G, k^*)$ (where k^* denotes the multiplicative group of non-zero elements of k). We show that by this isomorphism, $c(K_{R^G})$ corresponds to det, the determinant, in $\text{Hom}(G, k^*)$ and conclude the proof of Theorem 1.

We can apply Theorem 1 to the case of regular local rings. If (A, m) is a local ring and if $g\in \text{Aut}(A)$, g induces a linear transformation of the tangent space m/m^2 of A. We denote this correspondence by λ: $\text{Aut}(A)\to GL(m/m^2)$. We call an element g of $\text{Aut}(A)$ a pseudo-reflection if $\lambda(g)$ is a pseudo-reflection. Then, we have the following