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ON COMPLEX COBORDISM GROUPS OF CLASSIFYING SPACES FOR DIHEDRAL GROUPS

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1. Introduction

Let $G=H\cdot\Gamma$ be a semi-direct product of a finite group H by a finite group Γ, X a compact G-manifold which induces by restriction a principal H-manifold and Y a principal Γ-manifold. Then we have a principal G-space $X\times Y$ with a G-action defined by $h\gamma(x, y)=(h\gamma x, \gamma y)$, $h\gamma\in H\cdot\Gamma$. The equivariant map $i\colon X\to X\times Y$ defined by $i(x)=(x, y_0)$, induces a homomorphism

$$i^*\colon U^*((X\times Y)/G)\to U^*(X/H).$$

We can define a Γ-action over $U^*(X/H)$ corresponding to a Γ-action over the complex bordism group of unitary G-manifolds defined by (1.3) of [7]. The action is denoted by x^γ, $x\in U^*(X/H)$, $\gamma\in\Gamma$.

In this paper, we define a homomorphism

$$i_*\colon U^*(X/H)\to U^*((X\times Y)/G)$$

and obtain the following.

Theorem 1.1. *For* $x\in U^*(X/H)$, $i^*i_*(x)=\sum_{\gamma\in\Gamma}x^\gamma$.

Let $D_p(m, n)$ be the orbit manifold of $S^{2m+1}\times S^n$ by the dihedral group D_p whose action is given in [7]. Making use of Theorem 1.1 and the Atiyah-Hirzebruch spectral sequence of the complex cobordism group, we have the following.

Theorem 1.2. *Suppose that p is an odd prime. There exists an isomorphism*

$$\tilde U^{2m}(D_p(2k+1, 4k+3))\cong\tilde U^{2m}(L^{2k+1}(p))^{Z_2}\oplus\tilde U^{2m}(RP^{4k+3})\oplus U^{2m-8k-6},$$

where $L^l(p)=S^{2l+1}/Z_p$ *is a* $(2l+1)$*-dimensional lens space,* RP^s *is an s-dimensional real projective space and* $U^*(\ )^{Z_2}$ *is the subgroup consisting of the elements which are fixed under the* Z_2*-action.*

Let BZ_p be a classifying space for Z_p. There exists an isomorphism $U^{ev}(BZ_p)\cong U^*[[X]]/([p]_F(X))$, $U^{ev}(\ )=\sum U^{2i}(\ )$ [8]. Consider the Z_2-action on $U^{ev}(BZ_p)$ defined by