Ōshima, H.
Osaka J. Math.
**11** (1974), 361–366

# ON THE STABLE JAMES NUMBERS OF COMPLEX PROJECTIVE SPACES

HIDEAKI ŌSHIMA

(Received November 14, 1973)

## 1. Introduction

For a pointed finite $CW$-pair $i$: $A\subset X$ where $A$ is a connected oriented topological manifold, a (stable) map $f$: $X\to A$ is of type $r$ if the composite $A\xrightarrow{i}X\xrightarrow{f}A$ has degree $r$. $j(X, A)$ and $j_s(X, A)$ denote the sets of integers $r$ for which there exists a map $f$: $X\to A$ of type $r$ and a stable map of type $r$ respectively. When $j(X, A)$ forms an ideal $(k(X,A))$ in the ring of integers $Z$—here $k(X, A)$ denotes the non-negative generator, we call $k(X, A)$ the James number of the pair $(X, A)$. In the stable case $j_s(X, A)$ is always an ideal of $Z$. So we may define the stable James number $k_s(X, A)$.

James [3] has posed the problem of determining $j(SP^m(S^n), S^n)$, where $SP^m(S^n)$ is the $m$-fold symmetric product of an $n$-sphere $S^n$ with a base point $x_0$ and $i$: $S^n\to SP^m(S^n)$ is the axial embedding $x\to[x, x_0, \cdots, x_0]$. James showed for example $j(SP^m(S^n), S^n)$ forms an ideal of $Z$ and, for an even dimentional sphere $S^{2n}$, $k(SP^m(S^{2n}), S^{2n})=0$. On the contrary $k_s(SP^m(S^n), S^n)\neq 0$ for any positive integers $m$ and $n$. From now on we introduce the notation $k_s^{m,n}$ instead of $k_s(SP^m(S^n), S^n)$.

In this note we give lower bounds and an upper bound of $k_s^{m,2}$. That is, we prove

**Theorem.** *For positive integers m and n*

(1) $k_s^{m,n}\neq 0$;

(2) $k_s^{m+1,n}$ *is a multiple of* $k_s^{m,n}$;

(3) $k_s^{m,2}$ *is divisible by all the integers* $m, m-1, \cdots, 2$;

(4) $k_s^{2^m-1,2}$ *is divisible by* $2^m$ *for* $m\geqq 2$;

(5) $k_s^{m,2}$ *is a divisor of* $m!(m-1)!\cdots 2!$, *in particular none of the prime factors of* $k_s^{m,2}$ *is greater than m.*

**Corollary.** *The above lower estimates* (3) *and* (4) *are best possible for* $m\leqq 4$. *That is*

$$k_s^{1,2}=1,\ k_s^{2,2}=2,\ k_s^{3,2}=k_s^{4,2}=12\,.$$