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$[p]$-TYPICAL FORMAL GROUPS AND THE HOMOMORPHISM $\Omega_*^U \to \Omega_*^{SO}$

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In the present note we construct a $[p]$-typical formal group $F_{[p]}$, p a prime, which is universal for $[p]$-typical formal groups over arbitrary ground rings; and we study structure of the ground ring of $F_{[p]}$ (Corollary 5). Using $F_{[2]}$ we describe the kernel of the forgetful homomorphism S: $\Omega_*^U \to \Omega_*^{SO}$ of complex structures (Corollary 7).

Our basic reference is [1] and we use the notations of [1] freely.

1. Universal $[p]$-typical formal group

Let U be the Lazard ring and F_U the universal one-dimensional formal group defined over U. As usual we identify U with the complex cobordism ring Ω_*^U. Then U is graded by non-negative even degrees (or by non-positive even degrees when we regard Ω_*^U as $U^*(pt)$).

Let p be a prime, R a commutative ring with 1 and F a (commutative one-dimensional) formal group over R. By the terminology of [1] F is $[p]$-typical iff $\boldsymbol{f}_{p,F}\gamma_0=0$, where $\boldsymbol{f}_p$ is the Frobenius operator and γ_0 is the identity curve.

Let F be $[p]$-typical and u: $U \to R$ the unique unitary homomorphism of rings such that $u_* F_U = F$. By the notation of [1] we put

$$(\boldsymbol{f}_{p,U}\gamma_0)(T) = \sum_{n\geq 1}^{F_U} (v_{np-1}^{(p)} T^n) .$$

Then $v_{np-1}^{(p)} \in U_{2(np-1)}$. Now

$$u_*(\boldsymbol{f}_{p,U}\gamma_0) = \boldsymbol{f}_{p,F}\gamma_0 = 0 .$$

Hence

$$\sum_{n\geq 1}^{F} (u(v_{np-1}^{(p)}) T^n) = 0 ,$$

and by [1], Proposition 2.10, we obtain

(1.1) $$u(v_{np-1}^{(p)}) = 0 \qquad \text{for all } n \geq 1 .$$

Let

$$J_p = (v_{p-1}^{(p)}, v_{2p-1}^{(p)}, \cdots, v_{np-1}^{(p)}, \cdots) ,$$