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FINITE GROUPS WHICH ACT FREELY ON SPHERES

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We will study the problem: Let G be a finite group which acts freely (and topologically) on the sphere S^{2t-1}. Can G act freely and orthogonally on S^{2t-1}?

The result of T. Petrie [5] shows that the answer is no for t odd prime. As is easily seen, the answer is yes for $t=1$. The problem for $t=2$ is unsolved at present (see [2], [3], [4]). In this note it will be shown that the answer is yes for $t=4$, and also for $t=2^\nu$ ($\nu \geqq 3$) if G is solvable.

1. Preliminary theorems

By J. Milnor [3] we have

(1.1) *If G is a group which acts freely on S^n, then G satisfies the following properties*:

i) *Any element of order 2 in G belongs to the center of G.*

ii) *G has at most one element of order 2.*

The following (1.2) and (1.3) are shown in [1].

(1.2) *If G acts freely on S^n, the cohomology of G has period $n+1$.*

(1.3) *For a finite group G, the following two conditions are equivalent*:

i) *G has periodic cohomology.*

ii) *Every abelian subgroup of G is cyclic.*

A complete classification of finite groups satisfying the condition ii) of (1.3) is known by H. Zassenhaus [11] and M. Suzuki [6]. For future reference we reproduce it below after J. Wolf [10] and C.B. Thomas-C.T.C. Wall [8].

(1.4) *Let G be a finite group satisfying the condition* ii) *of* (1.3). *If G is solvable, it is one of the following groups*: